CHAPTER 3
Exponential and Logarithmic Functions

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Section 3.1 Exponential Functions and Their Graphs

- You should know that a function of the form $f(x) = a^x$, where $a > 0$, $a \neq 1$, is called an exponential function with base a.
- You should be able to graph exponential functions.
- You should know formulas for compound interest.

  (a) For n compoundings per year: $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$.

  (b) For continuous compoundings: $A = Pe^{rt}$.

Vocabulary Check

1. algebraic
2. transcendental
3. natural exponential; natural
4. $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$
5. $A = Pe^{rt}$

1. $f(5.6) = (3.4)^{5.6} \approx 946.852$

2. $f(x) = 2.3^x = 2.3^{3/2} \approx 3.488$

3. $f(-\pi) = 5^{-\pi} \approx 0.006$

4. $f(x) = \left(\frac{2}{3}\right)^{5x} = \left(\frac{2}{3}\right)^{5(0.3)} \approx 0.544$

5. $g(x) = 5000(2^x) = 5000(2^{-1.5})$

≈ 1767.767

6. $f(x) = 200(1.2)^{12x}$

$= 200(1.2)^{12 \cdot 24}$

$\approx 1.274 \times 10^{25}$

7. $f(x) = 2^x$

Increasing

Asymptote: $y = 0$

Intercept: $(0, 1)$

Matches graph (d).

8. $f(x) = 2^x + 1$ rises to the right.

Asymptote: $y = 1$

Intercept: $(0, 2)$

Matches graph (c).

9. $f(x) = 2^{-x}$

Decreasing

Asymptote: $y = 0$

Intercept: $(0, 1)$

Matches graph (a).

10. $f(x) = 2^{x-2}$ rises to the right.

Asymptote: $y = 0$

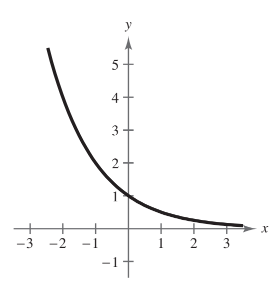
Intercept: $\left(0, \frac{1}{4}\right)$

Matches graph (b).

11. $f(x) = \left(\frac{1}{2}\right)^x$

x	-2	-1	0	1	2
$f(x)$	4	2	1	0.5	0.25

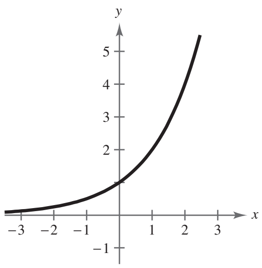
Asymptote: $y = 0$



12. $f(x) = \left(\frac{1}{2}\right)^{-x} = 2^x$

x	-2	-1	0	1	2
$f(x)$	0.25	0.5	1	2	4

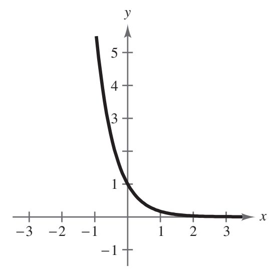
Asymptote: $y = 0$



13. $f(x) = 6^{-x}$

x	-2	-1	0	1	2
$f(x)$	36	6	1	0.167	0.028

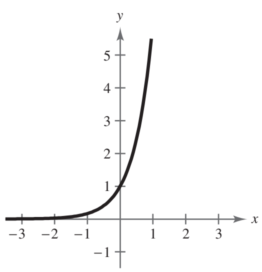
Asymptote: $y = 0$



14. $f(x) = 6^x$

x	-2	-1	0	1	2
$f(x)$	0.028	0.167	1	6	36

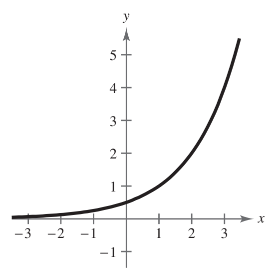
Asymptote: $y = 0$



15. $f(x) = 2^{x-1}$

x	-2	-1	0	1	2
$f(x)$	0.125	0.25	0.5	1	2

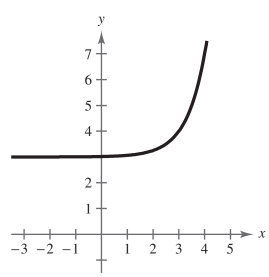
Asymptote: $y = 0$



16. $f(x) = 4^{x-3} + 3$

x	-1	0	1	2	3
$f(x)$	3.004	3.016	3.063	3.25	4

Asymptote: $y = 3$



17. $f(x) = 3^x$, $g(x) = 3^{x-4}$

Because $g(x) = f(x - 4)$, the graph of g can be obtained by shifting the graph of f four units to the right.

18. $f(x) = 4^x$, $g(x) = 4^x + 1$

Because $g(x) = f(x) + 1$, the graph of g can be obtained by shifting the graph of f one unit upward.

19. $f(x) = -2^x$, $g(x) = 5 - 2^x$

Because $g(x) = 5 + f(x)$, the graph of g can be obtained by shifting the graph of f five units upward.

20. $f(x) = 10^x$, $g(x) = 10^{-x+3}$

Because $g(x) = f(-x + 3)$, the graph of g can be obtained by reflecting the graph of f in the y-axis and shifting f three units to the right. (**Note:** This is equivalent to shifting f three units to the left and then reflecting the graph in the y-axis.)

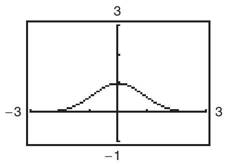
21. $f(x) = \left(\frac{7}{2}\right)^x,\ g(x) = -\left(\frac{7}{2}\right)^{-x+6}$

Because $g(x) = -f(-x + 6)$, the graph of g can be obtained by reflecting the graph of f in the x-axis and y-axis and shifting f six units to the right. (**Note:** This is equivalent to shifting f six units to the left and then reflecting the graph in the x-axis and y-axis.)

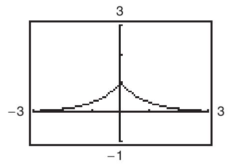
22. $f(x) = 0.3^x,\ g(x) = -0.3^x + 5$

$g(x) = -f(x) + 5$, hence the graph of g can be obtained by reflecting the graph of f in the x-axis and shifting the resulting graph five units upward.

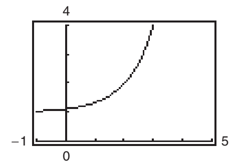
23. $y = 2^{-x^2}$



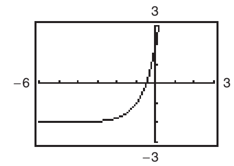
24. $y = 3^{-|x|}$



25. $f(x) = 3^{x-2} + 1$



26. $y = 4^{x+1} - 2$



27. $f\left(\frac{3}{4}\right) = e^{-3/4} \approx 0.472$

28. $f(x) = e^x = e^{3.2} \approx 24.533$

29. $f(10) = 2e^{-5(10)} \approx 3.857 \times 10^{-22}$

30. $f(x) = 1.5e^{(1/2)x}$
$= 1.5e^{120} \approx 1.956 \times 10^{52}$

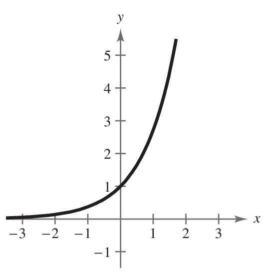
31. $f(6) = 5000e^{0.06(6)} \approx 7166.647$

32. $f(x) = 250e^{0.05x}$
$= 250e^{0.05(20)} \approx 679.570$

33. $f(x) = e^x$

x	-2	-1	0	1	2
$f(x)$	0.135	0.368	1	2.718	7.389

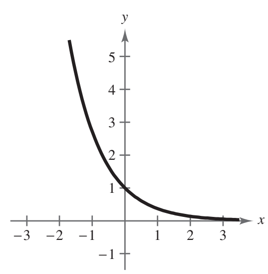
Asymptote: $y = 0$



34. $f(x) = e^{-x}$

x	-2	-1	0	1	2
$f(x)$	7.389	2.718	1	0.368	0.135

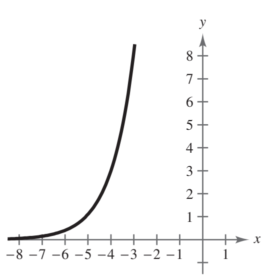
Asymptote: $y = 0$



35. $f(x) = 3e^{x+4}$

x	-8	-7	-6	-5	-4
$f(x)$	0.055	0.149	0.406	1.104	3

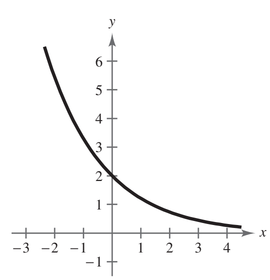
Asymptote: $y = 0$



36. $f(x) = 2e^{-0.5x}$

x	-2	-1	0	1	2
$f(x)$	5.437	3.297	2	1.213	0.736

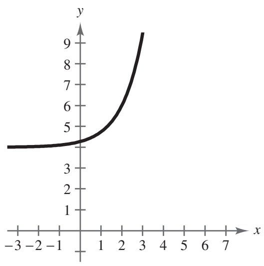
Asymptote: $y = 0$



37. $f(x) = 2e^{x-2} + 4$

x	-2	-1	0	1	2
$f(x)$	4.037	4.100	4.271	4.736	6

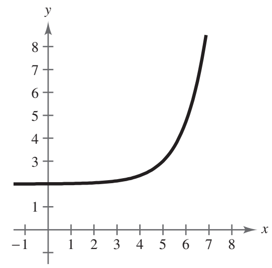
Asymptote: $y = 4$



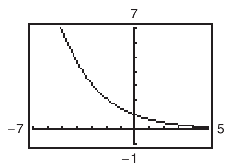
38. $f(x) = 2 + e^{x-5}$

x	0	2	4	5	6
$f(x)$	2.007	2.050	2.368	3	4.718

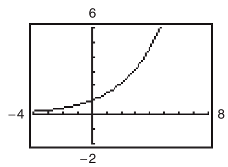
Asymptote: $y = 2$



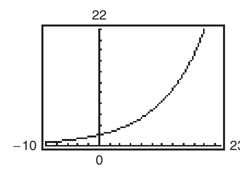
39. $y = 1.08^{-5x}$



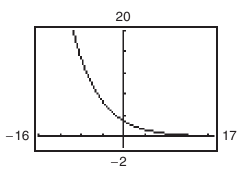
40. $y = 1.08^{5x}$



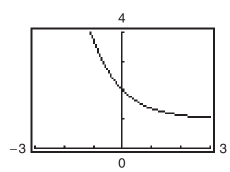
41. $s(t) = 2e^{0.12t}$



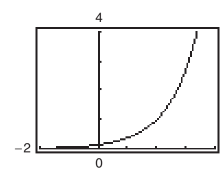
42. $s(t) = 3e^{-0.2t}$



43. $g(x) = 1 + e^{-x}$



44. $h(x) = e^{x-2}$



45. $3^{x+1} = 27$
$$3^{x+1} = 3^3$$
$$x + 1 = 3$$
$$x = 2$$

46. $2^{x-3} = 16$
$$2^{x-3} = 2^4$$
$$x - 3 = 4$$
$$x = 7$$

47. $2^{x-2} = \frac{1}{32}$
$$2^{x-2} = 2^{-5}$$
$$x - 2 = -5$$
$$x = -3$$

48. $\left(\frac{1}{5}\right)^{x+1} = 125$
$$\left(\frac{1}{5}\right)^{x+1} = 5^3$$
$$\left(\frac{1}{5}\right)^{x+1} = \left(\frac{1}{5}\right)^{-3}$$
$$x + 1 = -3$$
$$x = -4$$

49. $e^{3x+2} = e^3$
$$3x + 2 = 3$$
$$3x = 1$$
$$x = \frac{1}{3}$$

50. $e^{2x-1} = e^4$
$$2x - 1 = 4$$
$$2x = 5$$
$$x = \frac{5}{2}$$

51. $e^{x^2-3} = e^{2x}$
$$x^2 - 3 = 2x$$
$$x^2 - 2x - 3 = 0$$
$$(x - 3)(x + 1) = 0$$
$$x = 3 \text{ or } x = -1$$

52. $e^{x^2+6} = e^{5x}$
$$x^2 + 6 = 5x$$
$$x^2 - 5x + 6 = 0$$
$$(x - 3)(x - 2) = 0$$
$$x = 3 \text{ or } x = 2$$

53. $P = \$2500,\ r = 2.5\%,\ t = 10$ years

Compounded n times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 2500\left(1 + \frac{0.025}{n}\right)^{10n}$

Compounded continuously: $A = Pe^{rt} = 2500e^{0.025(10)}$

n	1	2	4	12	365	Continuous Compounding
A	\$3200.21	\$3205.09	\$3207.57	\$3209.23	\$3210.04	\$3210.06

54. $P = \$1000, r = 4\%, t = 10$ years

Compounded n times per year: $A = 1000\left(1 + \frac{0.04}{n}\right)^{10n}$

Compounded continuously: $A = 1000e^{0.04(10)}$

n	1	2	4	12	365	Continuous Compounding
A	\$1480.24	\$1485.95	\$1488.86	\$1490.83	\$1491.79	\$1491.82

55. $P = \$2500,\ r = 3\%,\ t = 20$ years

Compounded n times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 2500\left(1 + \frac{0.03}{n}\right)^{20n}$

Compounded continuously: $A = Pe^{rt} = 2500e^{0.03(20)}$

n	1	2	4	12	365	Continuous Compounding
A	\$4515.28	\$4535.05	\$4545.11	\$4551.89	\$4555.18	\$4555.30

56. $P = \$1000, r = 6\%, t = 40$ years

Compounded n times per year: $A = 1000\left(1 + \frac{0.06}{n}\right)^{40n}$

Compounded continuously: $A = 1000e^{0.06(40)}$

n	1	2	4	12	365	Continuous Compounding
A	\$10,285.72	\$10,640.89	\$10,828.46	\$10,957.45	\$11,021.00	\$11,023.18

57. $A = Pe^{rt} = 12{,}000e^{0.04t}$

t	10	20	30	40	50
A	\$17,901.90	\$26,706.49	\$39,841.40	\$59,436.39	\$88,668.67

58. $A = Pe^{rt} = 12{,}000e^{0.06t}$

t	10	20	30	40	50
A	\$21,865.43	\$39,841.40	\$72,595.77	\$132,278.12	\$241,026.44

59. $A = Pe^{rt} = 12{,}000e^{0.065t}$

t	10	20	30	40	50
A	$22,986.49	$44,031.56	$84,344.25	$161,564.86	$309,484.08

60. $A = Pe^{rt} = 12{,}000e^{0.035t}$

t	10	20	30	40	50
A	$17,028.81	$24,165.03	$34,291.81	$48,662.40	$69,055.23

61. $A = 25{,}000e^{(0.0875)(25)}$

$\approx \$222{,}822.57$

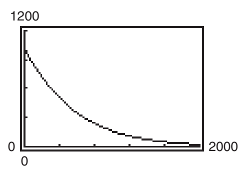
62. $A = 5000e^{(0.075)(50)}$

$\approx \$212{,}605.41$

63. $C(10) = 23.95(1.04)^{10} \approx \$35.45$

64. $p = 5000\left(1 - \dfrac{4}{4 + e^{-0.002x}}\right)$

(a)



(b) When $x = 500$:

$$p = 5000\left(1 - \frac{4}{4 + e^{-0.002(500)}}\right) \approx \$421.12$$

(c) Since $(600, 350.13)$ is on the graph in part (a), it appears that the greatest price that will still yield a demand of at least 600 units is about $350.

65. $V(t) = 100e^{4.6052t}$

(a) $V(1) \approx 10{,}000.298$ computers

(b) $V(1.5) \approx 10{,}004.472$ computers

(c) $V(2) \approx 1{,}000{,}059.63$ computers

66. (a) $P = 152.26e^{-0.0039t}$

Since the growth rate is negative, $-0.0039 = -0.39\%$, the population is decreasing.

(b) In 1998, $t = 8$ and the population is given by $P(8) = 152.26e^{-0.0039(8)} = 147.58$ million.

In 2000, $t = 10$ and the population is given by $P(10) = 152.26e^{-0.0039(10)} = 146.44$ million.

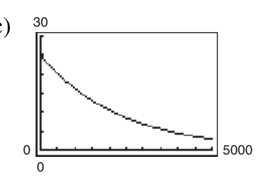
(c) In 2010, $t = 20$ and the population is given by $P(20) = 152.26e^{-0.0039(20)} = 140.84$ million.

67. $Q = 25\left(\frac{1}{2}\right)^{t/1599}$

(a) $Q(0) = 25$ grams

(b) $Q(1000) \approx 16.21$ grams

(c)



68. $Q = 10\left(\frac{1}{2}\right)^{t/5715}$

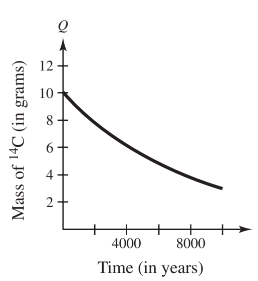
(a) When $t = 0$: $Q = 10\left(\frac{1}{2}\right)^{0/5715}$

$= 10(1) = 10$ grams

(b) When $t = 2000$: $Q = 10\left(\frac{1}{2}\right)^{2000/5715}$

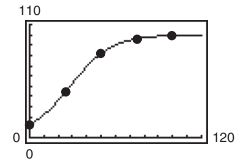
≈ 7.85 grams

(c)



69. $y = \dfrac{100}{1 + 7e^{-0.069x}}$

(a)



(b)

x	Sample Data	Model
0	12	12.5
25	44	44.5
50	81	81.82
75	96	96.19
100	99	99.3

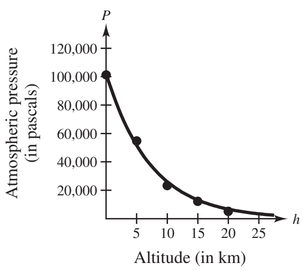
(c) When $x = 36$:

$$y = \frac{100}{1 + 7e^{-0.069(36)}} \approx 63.14\%.$$

(d) $\dfrac{2}{3}(100) = \dfrac{100}{1 + 7e^{-0.069x}}$ when

$x \approx 38$ masses.

70. (a)



(b) $p = 107{,}428e^{-0.150h}$

$= 107{,}428e^{-0.150(8)}$

$= 32{,}357$ pascals

71. True. The line $y = -2$ is a horizontal asymptote for the graph of $f(x) = 10^x - 2$.

72. False, $e \neq \frac{271{,}801}{99{,}990}$. e is an irrational number.

73. $f(x) = 3^{x-2}$

$= 3^x 3^{-2}$

$= 3^x\left(\dfrac{1}{3^2}\right)$

$= \dfrac{1}{9}(3^x)$

$= h(x)$

Thus, $f(x) \neq g(x)$, but $f(x) = h(x)$.

74. $g(x) = 2^{2x+6}$

$= 2^{2x} \cdot 2^6$

$= 64(2^{2x})$

$= 64(2^2)^x$

$= 64(4^x)$

$= h(x)$

Thus, $g(x) = h(x)$ but $g(x) \neq f(x)$.

75. $f(x) = 16(4^{-x})$ and $f(x) = 16(4^{-x})$

$= 4^2(4^{-x})$

$= 4^{2-x}$

$= \left(\frac{1}{4}\right)^{-(2-x)}$

$= \left(\frac{1}{4}\right)^{x-2}$

$= g(x)$

and

$= 16(2^2)^{-x}$

$= 16(2^{-2x})$

$= h(x)$

Thus, $f(x) = g(x) = h(x)$.

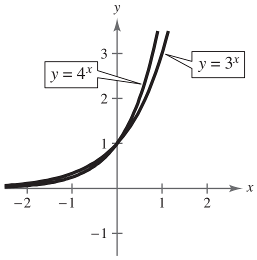
76. $f(x) = 5^{-x} + 3$

$g(x) = 5^{3-x} = 5^3 \cdot 5^{-x}$

$h(x) = -5^{x-3} = -(5^x \cdot 5^{-3})$

Thus, none are equal.

77. $y = 3^x$ and $y = 4^x$

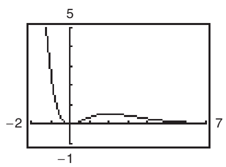


x	-2	-1	0	1	2
3^x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
4^x	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

(a) $4^x < 3^x$ when $x < 0$.

(b) $4^x > 3^x$ when $x > 0$.

78. (a) $f(x) = x^2e^{-x}$



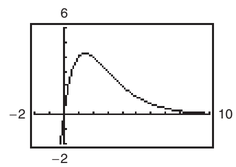
Decreasing: $(-\infty, 0)$, $(2, \infty)$

Increasing: $(0, 2)$

Relative maximum: $(2, 4e^{-2})$

Relative minimum: $(0, 0)$

(b) $g(x) = x2^{3-x}$

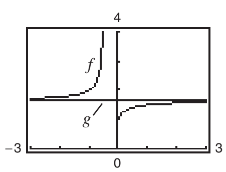


Decreasing: $(1.44, \infty)$

Increasing: $(-\infty, 1.44)$

Relative maximum: $(1.44, 4.25)$

79. $f(x) = \left(1 + \dfrac{0.5}{x}\right)^x$ and $g(x) = e^{0.5}$ (Horizontal line)



As $x \longrightarrow \infty$, $f(x) \longrightarrow g(x)$.

As $x \longrightarrow -\infty$, $f(x) \longrightarrow g(x)$.

80. The functions (c) 3^x and (d) 2^{-x} are exponential.

81. $x^2 + y^2 = 25$

$$y^2 = 25 - x^2$$

$$y = \pm\sqrt{25 - x^2}$$

82. $x - |y| = 2$

$$x - 2 = |y|$$

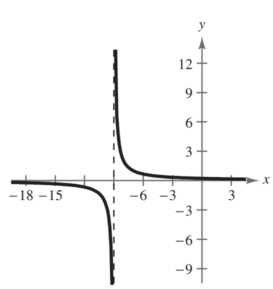
$$y = x - 2 \text{ and } y = -(x - 2),\ x \geq 2$$

83. $f(x) = \dfrac{2}{9 + x}$

Vertical asymptote: $x = -9$

Horizontal asymptote: $y = 0$

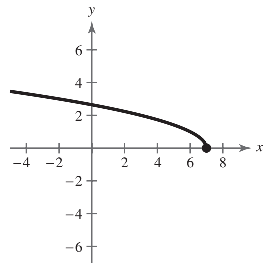
x	-11	-10	-8	-7
$f(x)$	-1	-2	2	1



84. $f(x) = \sqrt{7 - x}$

Domain: $(-\infty, 7]$

x	-9	-2	3	6	7
y	4	3	2	1	0



85. Answers will vary.

Section 3.2 Logarithmic Functions and Their Graphs

- You should know that a function of the form $y = \log_a x$, where $a > 0$, $a \neq 1$, and $x > 0$, is called a logarithm of x to base a.
- You should be able to convert from logarithmic form to exponential form and vice versa.

  $y = \log_a x \iff a^y = x$
- You should know the following properties of logarithms.
  (a) $\log_a 1 = 0$ since $a^0 = 1$.
  (b) $\log_a a = 1$ since $a^1 = a$.
  (c) $\log_a a^x = x$ since $a^x = a^x$.
  (d) $a^{\log_a x} = x$ Inverse Property
  (e) If $\log_a x = \log_a y$, then $x = y$.
- You should know the definition of the natural logarithmic function.

  $\log_e x = \ln x, x > 0$
- You should know the properties of the natural logarithmic function.
  (a) $\ln 1 = 0$ since $e^0 = 1$.
  (b) $\ln e = 1$ since $e^1 = e$.
  (c) $\ln e^x = x$ since $e^x = e^x$.
  (d) $e^{\ln x} = x$ Inverse Property
  (e) If $\ln x = \ln y$, then $x = y$.
- You should be able to graph logarithmic functions.

Vocabulary Check

1. logarithmic

2. 10

3. natural; e

4. $a^{\log_a x} = x$

5. $x = y$

1. $\log_4 64 = 3 \implies 4^3 = 64$

2. $\log_3 81 = 4 \implies 3^4 = 81$

3. $\log_7 \frac{1}{49} = -2 \implies 7^{-2} = \frac{1}{49}$

4. $\log \frac{1}{1000} = -3 \implies 10^{-3} = \frac{1}{1000}$

5. $\log_{32} 4 = \frac{2}{5} \implies 32^{2/5} = 4$

6. $\log_{16} 8 = \frac{3}{4} \implies 16^{3/4} = 8$

7. $\log_{36} 6 = \frac{1}{2} \implies 36^{1/2} = 6$

8. $\log_8 4 = \frac{2}{3} \implies 8^{2/3} = 4$

9. $5^3 = 125 \implies \log_5 125 = 3$

10. $8^2 = 64 \implies \log_8 64 = 2$

11. $81^{1/4} = 3 \implies \log_{81} 3 = \frac{1}{4}$

12. $9^{3/2} = 27 \implies \log_9 27 = \frac{3}{2}$

13. $6^{-2} = \frac{1}{36} \implies \log_6 \frac{1}{36} = -2$

14. $4^{-3} = \frac{1}{64} \implies \log_4 \frac{1}{64} = -3$

15. $7^0 = 1 \implies \log_7 1 = 0$

16. $10^{-3} = 0.001 \implies \log_{10} 0.001 = -3$

17. $f(x) = \log_2 x$

$f(16) = \log_2 16 = 4$ since $2^4 = 16$

18. $f(x) = \log_{16} x$

$f(4) = \log_{16} 4 = \frac{1}{2}$ since $16^{1/2} = 4$

19. $f(x) = \log_7 x$

$f(1) = \log_7 1 = 0$ since $7^0 = 1$

20. $f(x) = \log x$

$f(10) = \log 10 = 1$ since $10^1 = 10$

21. $g(x) = \log_a x$

$g(a^2) = \log_a a^2$

$= 2$ by the Inverse Property

22. $g(x) = \log_b x$

$g(b^{-3}) = \log_b b^{-3} = -3$ since

$b^{-3} = b^{-3}$

23. $f(x) = \log x$

$f\left(\frac{4}{5}\right) = \log\left(\frac{4}{5}\right) \approx -0.097$

24. $f(x) = \log x$

$f\left(\frac{1}{500}\right) = \log \frac{1}{500} \approx -2.699$

25. $f(x) = \log x$

$f(12.5) \approx 1.097$

26. $f(x) = \log x$

$f(75.25) \approx 1.877$

27. $\log_3 3^4 = 4$ since $3^4 = 3^4$

28. $\log_{1.5} 1$

Since $1.5^0 = 1$, $\log_{1.5} 1 = 0$.

29. $\log_\pi \pi = 1$ since $\pi^1 = \pi$.

30. $9^{\log_9 15}$

Since $a^{\log_a x} = x$, $9^{\log_9 15} = 15$.

31. $f(x) = \log_4 x$

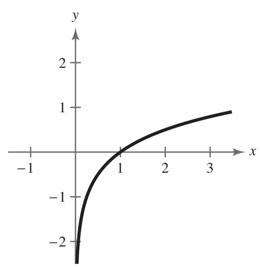
Domain: $x > 0 \implies$ The domain is $(0, \infty)$.

x-intercept: $(1, 0)$

Vertical asymptote: $x = 0$

$y = \log_4 x \implies 4^y = x$

x	$\frac{1}{4}$	1	4	2
$f(x)$	-1	0	1	$\frac{1}{2}$



32. $g(x) = \log_6 x$

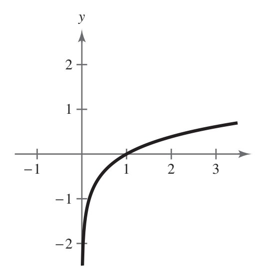
Domain: $(0, \infty)$

x-intercept: $(1, 0)$

Vertical asymptote: $x = 0$

$y = \log_6 x \implies 6^y = x$

x	$\frac{1}{6}$	1	$\sqrt{6}$	6
y	-1	0	$\frac{1}{2}$	1



33. $y = -\log_3 x + 2$

Domain: $(0, \infty)$

x-intercept:

$-\log_3 x + 2 = 0$

$2 = \log_3 x$

$3^2 = x$

$9 = x$

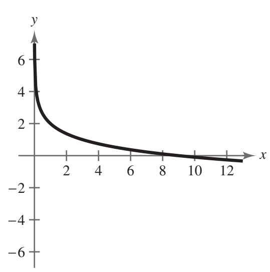
The x-intercept is $(9, 0)$.

Vertical asymptote: $x = 0$

$y = -\log_3 x + 2$

$\log_3 x = 2 - y \implies 3^{2-y} = x$

x	27	9	3	1	$\frac{1}{3}$
y	-1	0	1	2	3



34. $h(x) = \log_4(x - 3)$

Domain: $x - 3 > 0 \implies x > 3$

The domain is $(3, \infty)$.

x-intercept:

$\log_4(x - 3) = 0$

$4^0 = x - 3$

$1 = x - 3$

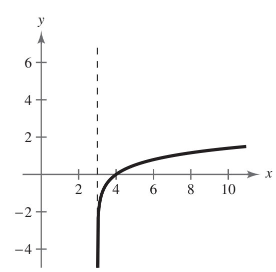
$4 = x$

The x-intercept is $(4, 0)$.

Vertical asymptote: $x - 3 = 0 \implies x = 3$

$y = \log_4(x - 3) \implies 4^y + 3 = x$

x	$3\frac{1}{4}$	4	7	19
y	-1	0	1	2



35. $f(x) = -\log_6(x + 2)$

Domain: $x + 2 > 0 \implies x > -2$

The domain is $(-2, \infty)$.

x-intercept:

$0 = -\log_6(x + 2)$

$0 = \log_6(x + 2)$

$6^0 = x + 2$

$1 = x + 2$

$-1 = x$

The x-intercept is $(-1, 0)$.

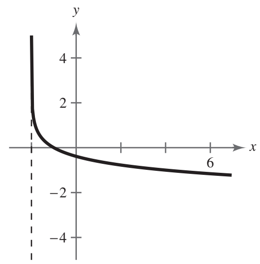
Vertical asymptote: $x + 2 = 0 \implies x = -2$

$y = -\log_6(x + 2)$

$-y = \log_6(x + 2)$

$6^{-y} - 2 = x$

x	4	-1	$-1\frac{5}{6}$	$-1\frac{35}{36}$
$f(x)$	-1	0	1	2



36. $y = \log_5(x - 1) + 4$

Domain: $x - 1 > 0 \implies x > 1$

The domain is $(1, \infty)$.

x-intercept:

$\log_5(x - 1) + 4 = 0$

$\log_5(x - 1) = -4$

$5^{-4} = x - 1$

$\frac{1}{625} = x - 1$

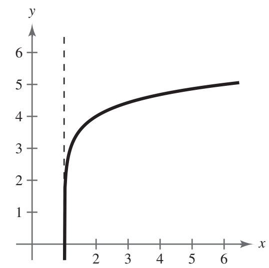
$\frac{626}{625} = x$

The x-intercept is $\left(\frac{626}{625}, 0\right)$.

Vertical asymptote: $x - 1 = 0 \implies x = 1$

$y = \log_5(x - 1) + 4 \implies 5^{y-4} + 1 = x$

x	1.00032	1.0016	1.008	1.04	1.2
y	-1	0	1	2	3



37. $y = \log\left(\dfrac{x}{5}\right)$

Domain: $\dfrac{x}{5} > 0 \implies x > 0$

The domain is $(0, \infty)$.

x-intercept:

$\log\left(\dfrac{x}{5}\right) = 0$

$\dfrac{x}{5} = 10^0$

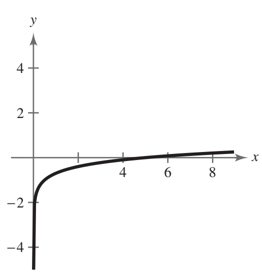
$\dfrac{x}{5} = 1 \implies x = 5$

The x-intercept is $(5, 0)$.

Vertical asymptote: $\dfrac{x}{5} = 0 \implies x = 0$

The vertical asymptote is the y-axis.

x	1	2	3	4	5	6	7
y	-0.70	-0.40	-0.22	-0.10	0	0.08	0.15



38. $y = \log(-x)$

Domain: $-x > 0 \implies x < 0$

The domain is $(-\infty, 0)$.

x-intercept: $\log(-x) = 0$

$10^0 = -x$

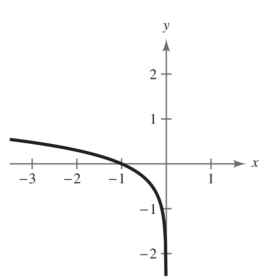
$-1 = x$

The x-intercept is $(-1, 0)$.

Vertical asymptote: $x = 0$

$y = \log(-x) \implies -10^y = x$

x	$-\frac{1}{100}$	$-\frac{1}{10}$	-1	-10
y	-2	-1	0	1



39. $f(x) = \log_3 x + 2$

Asymptote: $x = 0$

Point on graph: $(1, 2)$

Matches graph (c).

The graph of $f(x)$ is obtained by shifting the graph of $g(x)$ upward two units.

40. $f(x) = -\log_3 x$

Asymptote: $x = 0$

Point on graph: $(1, 0)$

Matches graph (f).

$f(x)$ reflects $g(x)$ in the x-axis.

41. $f(x) = -\log_3(x + 2)$

Asymptote: $x = -2$

Point on graph: $(-1, 0)$

Matches graph (d).

The graph of $f(x)$ is obtained by reflecting the graph of $g(x)$ about the x-axis and shifting the graph two units to the left.

42. $f(x) = \log_3(x - 1)$

Asymptote: $x = 1$

Point on graph: $(2, 0)$

Matches graph (e).

$f(x)$ shifts $g(x)$ one unit to the right.

43. $f(x) = \log_3(1 - x) = \log_3[-(x - 1)]$

Asymptote: $x = 1$

Point on graph: $(0, 0)$

Matches graph (b).

The graph of $f(x)$ is obtained by reflecting the graph of $g(x)$ about the y-axis and shifting the graph one unit to the right.

44. $f(x) = -\log_3(-x)$

Asymptote: $x = 0$

Point on graph: $(-1, 0)$

Matches graph (a).

$f(x)$ reflects $g(x)$ in the x-axis then reflects that graph in the y-axis.

45. $\ln \frac{1}{2} = -0.693\ldots \implies e^{-0.693\ldots} = \frac{1}{2}$

46. $\ln \frac{2}{5} = -0.916\ldots \implies e^{-0.916\ldots} = \frac{2}{5}$

47. $\ln 4 = 1.386\ldots \implies e^{1.386\ldots} = 4$

48. $\ln 10 = 2.302\ldots \implies e^{2.302\ldots} = 10$

49. $\ln 250 = 5.521\ldots \implies e^{5.521\ldots} = 250$

50. $\ln 679 = 6.520\ldots \implies e^{6.520\ldots} = 679$

51. $\ln 1 = 0 \implies e^0 = 1$

52. $\ln e = 1 \implies e^1 = e$

53. $e^3 = 20.0855\ldots \implies \ln 20.0855\ldots = 3$

54. $e^2 = 7.3890\ldots \implies \ln 7.3890\ldots = 2$

55. $e^{1/2} = 1.6487\ldots \implies \ln 1.6487\ldots = \frac{1}{2}$

56. $e^{1/3} = 1.3956\ldots \implies \ln 1.3956\ldots = \frac{1}{3}$

57. $e^{-0.5} = 0.6065\ldots \implies \ln 0.6065\ldots = -0.5$

58. $e^{-4.1} = 0.0165\ldots \implies \ln 0.0165\ldots = -4.1$

59. $e^x = 4 \implies \ln 4 = x$

60. $e^{2x} = 3 \implies \ln 3 = 2x$

61. $f(x) = \ln x$

$f(18.42) = \ln 18.42 \approx 2.913$

62. $f(x) = 3 \ln x$

$f(0.32) = 3 \ln 0.32 \approx -3.418$

63. $g(x) = 2 \ln x$

$g(0.75) = 2 \ln 0.75 \approx -0.575$

64. $g(x) = -\ln x$

$g\left(\frac{1}{2}\right) = -\ln \frac{1}{2} \approx 0.693$

65. $g(x) = \ln x$

$g(e^3) = \ln e^3 = 3$ by the Inverse Property

66. $g(x) = \ln x$

$g(e^{-2}) = \ln e^{-2} = -2$

67. $g(x) = \ln x$

$g(e^{-2/3}) = \ln e^{-2/3} = -\frac{2}{3}$ by the Inverse Property

68. $g(x) = \ln x$

$g(e^{-5/2}) = \ln e^{-5/2} = -\frac{5}{2}$

69. $f(x) = \ln(x - 1)$

Domain: $x - 1 > 0 \implies x > 1$

The domain is $(1, \infty)$.

x-intercept:

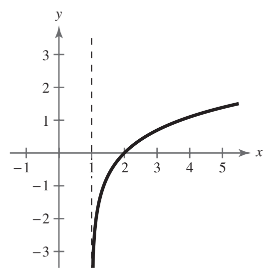
$0 = \ln(x - 1)$

$e^0 = x - 1$

$2 = x$

The x-intercept is $(2, 0)$.

Vertical asymptote: $x - 1 = 0 \implies x = 1$



x	1.5	2	3	4
$f(x)$	−0.69	0	0.69	1.10

70. $h(x) = \ln(x + 1)$

Domain: $x + 1 > 0 \implies x > -1$

The domain is $(-1, \infty)$.

x-intercept:

$\ln(x + 1) = 0$

$e^0 = x + 1$

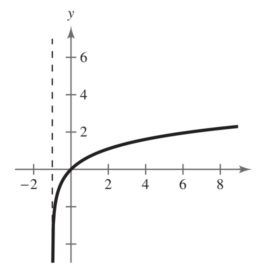
$1 = x + 1$

$0 = x$

The x-intercept is $(0, 0)$.

Vertical asymptote: $x + 1 = 0 \implies x = -1$

$y = \ln(x + 1) \implies e^y - 1 = x$



x	−0.39	0	1.72	6.39	19.09
y	$-\frac{1}{2}$	0	1	2	3

71. $g(x) = \ln(-x)$

Domain: $-x > 0 \implies x < 0$

The domain is $(-\infty, 0)$.

x-intercept:

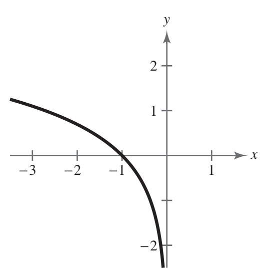
$0 = \ln(-x)$

$e^0 = -x$

$-1 = x$

The x-intercept is $(-1, 0)$.

Vertical asymptote: $-x = 0 \implies x = 0$



x	−0.5	−1	−2	−3
$g(x)$	−0.69	0	0.69	1.10

72. $f(x) = \ln(3 - x)$

Domain: $3 - x > 0 \implies x < 3$

The domain is $(-\infty, 3)$.

x-intercept:

$\ln(3 - x) = 0$

$e^0 = 3 - x$

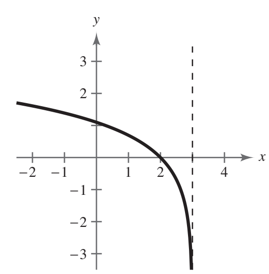
$1 = 3 - x$

$2 = x$

The x-intercept is $(2, 0)$.

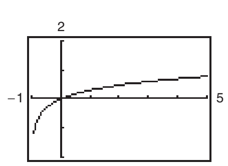
Vertical asymptote: $3 - x = 0 \implies x = 3$

$y = \ln(3 - x) \implies 3 - e^y = x$

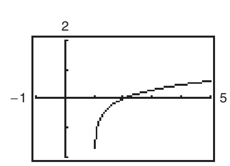


x	2.95	2.86	2.63	2	0.28
y	−3	−2	−1	0	1

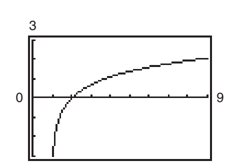
73. $y_1 = \log(x + 1)$



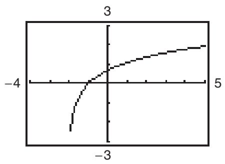
74. $f(x) = \log(x - 1)$



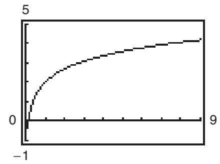
75. $y_1 = \ln(x - 1)$



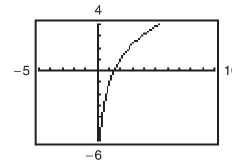
76. $f(x) = \ln(x + 2)$



77. $y = \ln x + 2$



78. $f(x) = 3 \ln x - 1$



79. $\log_2(x + 1) = \log_2 4$

$$x + 1 = 4$$
$$x = 3$$

80. $\log_2(x - 3) = \log_2 9$

$$x - 3 = 9$$
$$x = 12$$

81. $\log(2x + 1) = \log 15$

$$2x + 1 = 15$$
$$x = 7$$

82. $\log(5x + 3) = \log 12$

$$5x + 3 = 12$$
$$5x = 9$$
$$x = \tfrac{9}{5}$$

83. $\ln(x + 2) = \ln 6$

$$x + 2 = 6$$
$$x = 4$$

84. $\ln(x - 4) = \ln 2$

$$x - 4 = 2$$
$$x = 6$$

85. $\ln(x^2 - 2) = \ln 23$

$$x^2 - 2 = 23$$
$$x^2 = 25$$
$$x = \pm 5$$

86. $\ln(x^2 - x) = \ln 6$

$$x^2 - x = 6$$
$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$$x = -2 \text{ or } x = 3$$

87. $t = 12.542 \ln\left(\dfrac{x}{x - 1000}\right), x > 1000$

(a) When $x = \$1100.65$:

$$t = 12.542 \ln\left(\frac{1100.65}{1100.65 - 1000}\right) \approx 30 \text{ years}$$

When $x = \$1254.68$:

$$t = 12.542 \ln\left(\frac{1254.68}{1254.68 - 1000}\right) \approx 20 \text{ years}$$

(b) Total amounts: $(1100.65)(12)(30) = \$396{,}234.00$

$(1254.68)(12)(20) = \$301{,}123.20$

(c) Interest charges: $396{,}234 - 150{,}000 = \$246{,}234$

$301{,}123.20 - 150{,}000 = \$151{,}123.20$

(d) The vertical asymptote is $x = 1000$. The closer the payment is to \$1000 per month, the longer the length of the mortgage will be. Also, the monthly payment must be greater than \$1000.

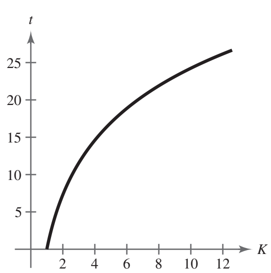
88. $t = \dfrac{\ln K}{0.095}$

(a)

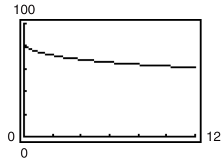
K	1	2	4	6	8	10	12
t	0	7.3	14.6	18.9	21.9	24.2	26.2

The number of years required to multiply the original investment by K increases with K. However, the larger the value of K, the fewer the years required to increase the value of the investment by an additional multiple of the original investment.

(b)



89. $f(t) = 80 - 17\log(t + 1), 0 \le t \le 12$

(a) 

(b) $f(0) = 80 - 17 \log 1 = 80.0$

(c) $f(4) = 80 - 17 \log 5 \approx 68.1$

(d) $f(10) = 80 - 17 \log 11 \approx 62.3$

90. $\beta = 10 \log\left(\dfrac{I}{10^{-12}}\right)$

(a) $\beta = 10 \log\left(\dfrac{1}{10^{-12}}\right) = 10 \log(10^{12}) = 120$ decibels

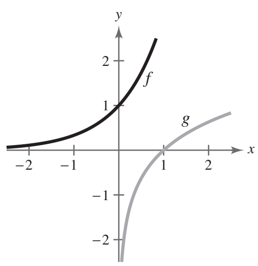
(b) $\beta = 10 \log\left(\dfrac{10^{-2}}{10^{-12}}\right) = 10 \log(10^{10}) = 100$ decibels

(c) No, the difference is due to the logarithmic relationship between intensity and number of decibels.

91. False. Reflecting $g(x)$ about the line $y = x$ will determine the graph of $f(x)$.

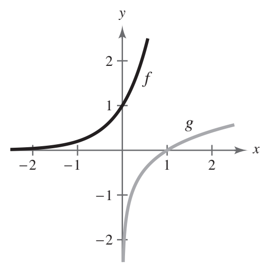
92. True, $\log_3 27 = 3 \implies 3^3 = 27$.

93. $f(x) = 3^x$, $g(x) = \log_3 x$



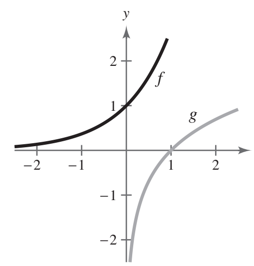
f and g are inverses. Their graphs are reflected about the line $y = x$.

94. $f(x) = 5^x$, $g(x) = \log_5 x$



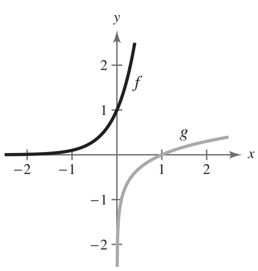
f and g are inverses. Their graphs are reflected about the line $y = x$.

95 . $f(x) = e^x$, $g(x) = \ln x$



f and g are inverses. Their graphs are reflected about the line $y = x$.

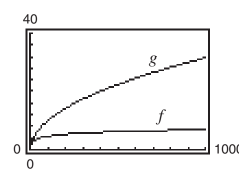
96. $f(x) = 10^x$, $g(x) = \log_{10} x$



f and g are inverses. Their graphs are reflected about the line $y = x$.

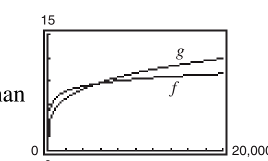
97. (a) $f(x) = \ln x$, $g(x) = \sqrt{x}$

The natural log function grows at a slower rate than the square root function.



(b) $f(x) = \ln x$, $g(x) = \sqrt[4]{x}$

The natural log function grows at a slower rate than the fourth root function.

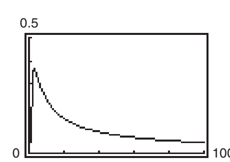


98. $f(x) = \dfrac{\ln x}{x}$

(a)

x	1	5	10	10^2	10^4	10^6
$f(x)$	0	0.322	0.230	0.046	0.00092	0.0000138

(b) As $x \to \infty$, $f(x) \to 0$.

(c) 

99. (a) False. If y were an exponential function of x, then $y = a^x$, but $a^1 = a$, not 0. Because one point is $(1, 0)$, y is not an exponential function of x.

(b) True. $y = \log_a x$

For $a = 2$, $y = \log_2 x$.

$x = 1, \log_2 1 = 0$

$x = 2, \log_2 2 = 1$

$x = 8, \log_2 8 = 3$

(c) True. $x = a^y$

For $a = 2$, $x = 2^y$.

$y = 0, 2^0 = 1$

$y = 1, 2^1 = 2$

$y = 3, 2^3 = 8$

(d) False. If y were a linear function of x, the slope between $(1, 0)$ and $(2, 1)$ and the slope between $(2, 1)$ and $(8, 3)$ would be the same. However,

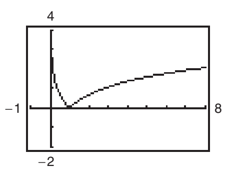
$$m_1 = \frac{1 - 0}{2 - 1} = 1 \text{ and } m_2 = \frac{3 - 1}{8 - 2} = \frac{2}{6} = \frac{1}{3}.$$

Therefore, y is not a linear function of x.

100. $y = \log_a x \implies a^y = x$, so, for example, if $a = -2$, there is no value of y for which $(-2)^y = -4$. If $a = 1$, then every power of a is equal to 1, so x could only be 1. So, $\log_a x$ is defined only for $0 < a < 1$ and $a > 1$.

101. $f(x) = |\ln x|$

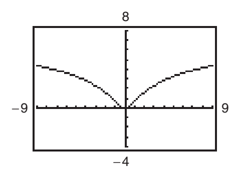
(a)



(b) Increasing on $(1, \infty)$
Decreasing on $(0, 1)$

(c) Relative minimum: $(1, 0)$

102. (a) $h(x) = \ln(x^2 + 1)$



(b) Increasing on $(0, \infty)$
Decreasing on $(-\infty, 0)$

(c) Relative minimum: $(0, 0)$

For Exercises 103–108, use $f(x) = 3x + 2$ and $g(x) = x^3 - 1$.

103.
$$\begin{aligned}(f + g)(2) &= f(2) + g(2)\\ &= [3(2) + 2] + [(2)^3 - 1]\\ &= 8 + 7\\ &= 15\end{aligned}$$

104.
$$\begin{aligned}f(x) - g(x) &= 3x + 2 - (x^3 - 1)\\ &= 3x + 2 - x^3 + 1\\ &= 3x - x^3 + 3\end{aligned}$$

Therefore,

$$\begin{aligned}(f - g)(-1) &= 3(-1) - (-1)^3 + 3\\ &= -3 + 1 + 3\\ &= 1.\end{aligned}$$

105.
$$\begin{aligned}(fg)(6) &= f(6)g(6)\\ &= [3(6) + 2][(6)^3 - 1]\\ &= (20)(215)\\ &= 4300\end{aligned}$$

106. $\dfrac{f(x)}{g(x)} = \dfrac{3x + 2}{x^3 - 1}$

Therefore, $\left(\dfrac{f}{g}\right)(0) = \dfrac{3 \cdot 0 + 2}{0^3 - 1} = -2.$

107.
$$\begin{aligned}(f \circ g)(7) &= f(g(7))\\ &= f((7)^3 - 1)\\ &= f(342)\\ &= 3(342) + 2\\ &= 1028\end{aligned}$$

108. $(g \circ f)(x) = g(f(x)) = g(3x + 2) = (3x + 2)^3 - 1$

Therefore,

$$\begin{aligned}(g \circ f)(-3) &= (3 \cdot (-3) + 2)^3 - 1\\ &= -7^3 - 1 = -344.\end{aligned}$$

Section 3.3 Properties of Logarithms

- You should know the following properties of logarithms.
  (a) $\log_a x = \dfrac{\log_b x}{\log_b a}$ $\quad \log_a x = \dfrac{\log_{10} x}{\log_{10} a}$ $\quad \log_a x = \dfrac{\ln x}{\ln a}$
  (b) $\log_a(uv) = \log_a u + \log_a v$ $\quad \ln(uv) = \ln u + \ln v$
  (c) $\log_a\left(\dfrac{u}{v}\right) = \log_a u - \log_a v$ $\quad \ln\left(\dfrac{u}{v}\right) = \ln u - \ln v$
  (d) $\log_a u^n = n \log_a u$ $\quad \ln u^n = n \ln u$
- You should be able to rewrite logarithmic expressions using these properties.

Vocabulary Check

1. change-of-base

2. $\dfrac{\log x}{\log a} = \dfrac{\ln x}{\ln a}$

3. $\log_a(uv) = \log_a u + \log_a v$
This is the Product Property. Matches (c).

4. $\ln u^n = n \ln u$
This is the Power Property. Matches (a).

5. $\log_a \dfrac{u}{v} = \log_a u - \log_a v$
This is the Quotient Property. Matches (b).

1. (a) $\log_5 x = \dfrac{\log x}{\log 5}$
(b) $\log_5 x = \dfrac{\ln x}{\ln 5}$

2. (a) $\log_3 x = \dfrac{\log x}{\log 3}$
(b) $\log_3 x = \dfrac{\ln x}{\ln 3}$

3. (a) $\log_{1/5} x = \dfrac{\log x}{\log(1/5)}$
(b) $\log_{1/5} x = \dfrac{\ln x}{\ln(1/5)}$

4. (a) $\log_{1/3} x = \dfrac{\log x}{\log(1/3)}$
(b) $\log_{1/3} x = \dfrac{\ln x}{\ln(1/3)}$

5. (a) $\log_x \dfrac{3}{10} = \dfrac{\log(3/10)}{\log x}$
(b) $\log_x \dfrac{3}{10} = \dfrac{\ln(3/10)}{\ln x}$

6. (a) $\log_x \dfrac{3}{4} = \dfrac{\log(3/4)}{\log x}$
(b) $\log_x \dfrac{3}{4} = \dfrac{\ln(3/4)}{\ln x}$

7. (a) $\log_{2.6} x = \dfrac{\log x}{\log 2.6}$
(b) $\log_{2.6} x = \dfrac{\ln x}{\ln 2.6}$

8. (a) $\log_{7.1} x = \dfrac{\log x}{\log 7.1}$
(b) $\log_{7.1} x = \dfrac{\ln x}{\ln 7.1}$

9. $\log_3 7 = \dfrac{\log 7}{\log 3} = \dfrac{\ln 7}{\ln 3} \approx 1.771$

10. $\log_7 4 = \dfrac{\log 4}{\log 7} = \dfrac{\ln 4}{\ln 7} \approx 0.712$

11. $\log_{1/2} 4 = \dfrac{\log 4}{\log(1/2)} = \dfrac{\ln 4}{\ln(1/2)} = -2.000$

12. $\log_{1/4} 5 = \dfrac{\log 5}{\log(1/4)} = \dfrac{\ln 5}{\ln(1/4)} \approx -1.161$

13. $\log_9(0.4) = \dfrac{\log 0.4}{\log 9} = \dfrac{\ln 0.4}{\ln 9} \approx -0.417$

14. $\log_{20} 0.125 = \dfrac{\log 0.125}{\log 20} = \dfrac{\ln 0.125}{\ln 20} \approx -0.694$

15. $\log_{15} 1250 = \dfrac{\log 1250}{\log 15} = \dfrac{\ln 1250}{\ln 15} \approx 2.633$

16. $\log_3 0.015 = \dfrac{\log 0.015}{\log 3} = \dfrac{\ln 0.015}{\ln 3} \approx -3.823$

17. $\log_4 8 = \dfrac{\log_2 8}{\log_2 4} = \dfrac{\log_2 2^3}{\log_2 2^2} = \dfrac{3}{2}$

18. $\log_2(4^2 \cdot 3^4) = \log_2 4^2 + \log_2 3^4$
$= 2\log_2 4 + 4\log_2 3$
$= 2\log_2 2^2 + 4\log_2 3$
$= 4\log_2 2 + 4\log_2 3$
$= 4 + 4\log_2 3$

19. $\log_5 \frac{1}{250} = \log_5\left(\frac{1}{125} \cdot \frac{1}{2}\right)$
$= \log_5 \frac{1}{125} + \log_5 \frac{1}{2}$
$= \log_5 5^{-3} + \log_5 2^{-1}$
$= -3 - \log_5 2$

20. $\log \frac{9}{300} = \log \frac{3}{100}$
$= \log 3 - \log 100$
$= \log 3 - \log 10^2$
$= \log 3 - 2\log 10$
$= \log 3 - 2$

21. $\ln(5e^6) = \ln 5 + \ln e^6$
$= \ln 5 + 6$
$= 6 + \ln 5$

22. $\ln \dfrac{6}{e^2} = \ln 6 - \ln e^2$
$= \ln 6 - 2\ln e$
$= \ln 6 - 2$

23. $\log_3 9 = 2\log_3 3 = 2$

24. $\log_5 \frac{1}{125} = \log_5 5^{-3} = -3\log_5 5 = -3(1) = -3$

25. $\log_2 \sqrt[4]{8} = \frac{1}{4}\log_2 2^3 = \frac{3}{4}\log_2 2 = \frac{3}{4}(1) = \frac{3}{4}$

26. $\log_6 \sqrt[3]{6} = \log_6 6^{1/3} = \frac{1}{3}\log_6 6 = \frac{1}{3}(1) = \frac{1}{3}$

27. $\log_4 16^{1.2} = 1.2(\log_4 16) = 1.2\log_4 4^2 = 1.2(2) = 2.4$

28. $\log_3 81^{-0.2} = -0.2\log_3 81$
$= -0.2\log_3 3^4$
$= -0.2(4) = -0.8$

29. $\log_3(-9)$ is undefined. -9 is not in the domain of $\log_3 x$.

30. $\log_2(-16)$ is undefined because -16 is not in the domain of $\log_2 x$.

31. $\ln e^{4.5} = 4.5$

32. $3\ln e^4 = (3)(4)\ln e$
$= 12(1)$
$= 12$

33. $\ln \dfrac{1}{\sqrt{e}} = \ln 1 - \ln\sqrt{e}$
$= 0 - \dfrac{1}{2}\ln e$
$= 0 - \dfrac{1}{2}(1)$
$= -\dfrac{1}{2}$

34. $\ln \sqrt[4]{e^3} = \ln e^{3/4}$
$= \dfrac{3}{4}\ln e$
$= \dfrac{3}{4}(1)$
$= \dfrac{3}{4}$

35. $\ln e^2 + \ln e^5 = 2 + 5 = 7$

36. $2\ln e^6 - \ln e^5 = \ln e^{12} - \ln e^5$
$= \ln \dfrac{e^{12}}{e^5}$
$= \ln e^7$
$= 7$

37. $\log_5 75 - \log_5 3 = \log_5 \dfrac{75}{3}$
$= \log_5 25$
$= \log_5 5^2$
$= 2\log_5 5$
$= 2$

38. $\log_4 2 + \log_4 32 = \log_4 4^{1/2} + \log_4 4^{5/2}$
$= \frac{1}{2}\log_4 4 + \frac{5}{2}\log_4 4$
$= \frac{1}{2}(1) + \frac{5}{2}(1)$
$= 3$

39. $\log_4 5x = \log_4 5 + \log_4 x$

40. $\log_3 10z = \log_3 10 + \log_3 z$

41. $\log_8 x^4 = 4 \log_8 x$

42. $\log \dfrac{y}{2} = \log y - \log 2$

43. $\log_5 \dfrac{5}{x} = \log_5 5 - \log_5 x$

$= 1 - \log_5 x$

44. $\log_6 z^{-3} = -3 \log_6 z$

45. $\ln \sqrt{z} = \ln z^{1/2} = \dfrac{1}{2} \ln z$

46. $\ln \sqrt[3]{t} = \ln t^{1/3} = \frac{1}{3} \ln t$

47. $\ln xyz^2 = \ln x + \ln y + \ln z^2$

$= \ln x + \ln y + 2 \ln z$

48. $\log 4x^2y = \log 4 + \log x^2 + \log y$

$= \log 4 + 2 \log x + \log y$

49. $\ln z(z-1)^2 = \ln z + \ln(z-1)^2$

$= \ln z + 2\ln(z-1), \ z > 1$

50. $\ln\left(\dfrac{x^2-1}{x^3}\right) = \ln(x^2-1) - \ln x^3$

$= \ln[(x+1)(x-1)] - \ln x^3$

$= \ln(x+1) + \ln(x-1) - 3 \ln x$

51. $\log_2 \dfrac{\sqrt{a-1}}{9} = \log_2 \sqrt{a-1} - \log_2 9$

$= \dfrac{1}{2} \log_2(a-1) - \log_2 3^2$

$= \dfrac{1}{2} \log_2(a-1) - 2 \log_2 3, \ a > 1$

52. $\ln \dfrac{6}{\sqrt{x^2+1}} = \ln 6 - \ln \sqrt{x^2+1}$

$= \ln 6 - \ln(x^2+1)^{1/2}$

$= \ln 6 - \dfrac{1}{2} \ln(x^2+1)$

53. $\ln \sqrt[3]{\dfrac{x}{y}} = \dfrac{1}{3} \ln \dfrac{x}{y}$

$= \dfrac{1}{3}[\ln x - \ln y]$

$= \dfrac{1}{3} \ln x - \dfrac{1}{3} \ln y$

54. $\ln \sqrt{\dfrac{x^2}{y^3}} = \ln\left(\dfrac{x^2}{y^3}\right)^{1/2} = \dfrac{1}{2} \ln\left(\dfrac{x^2}{y^3}\right)$

$= \dfrac{1}{2}(\ln x^2 - \ln y^3)$

$= \dfrac{1}{2}(2 \ln x - 3 \ln y)$

$= \ln x - \dfrac{3}{2} \ln y$

55. $\ln\left(\dfrac{x^4\sqrt{y}}{z^5}\right) = \ln x^4\sqrt{y} - \ln z^5$

$= \ln x^4 + \ln \sqrt{y} - \ln z^5$

$= 4 \ln x + \dfrac{1}{2} \ln y - 5 \ln z$

56. $\log_2 \dfrac{\sqrt{x}\,y^4}{z^4} = \log_2 \sqrt{x}\,y^4 - \log_2 z^4$

$= \log_2 \sqrt{x} + \log_2 y^4 - \log_2 z^4$

$= \dfrac{1}{2} \log_2 x + 4 \log_2 y - 4 \log_2 z$

57. $\log_5\left(\dfrac{x^2}{y^2z^3}\right) = \log_5 x^2 - \log_5 y^2z^3$

$= \log_5 x^2 - (\log_5 y^2 + \log_5 z^3)$

$= 2 \log_5 x - 2 \log_5 y - 3 \log_5 z$

58. $\log \dfrac{xy^4}{z^5} = \log xy^4 - \log z^5$

$= \log x + \log y^4 - \log z^5$

$= \log x + 4 \log y - 5 \log z$

59. $\ln \sqrt[4]{x^3(x^2+3)} = \frac{1}{4} \ln x^3(x^2+3)$

$= \frac{1}{4}[\ln x^3 + \ln(x^2+3)]$

$= \frac{1}{4}[3 \ln x + \ln(x^2+3)]$

$= \frac{3}{4} \ln x + \frac{1}{4} \ln(x^2+3)$

60. $\ln \sqrt{x^2(x+2)} = \ln[x^2(x+2)]^{1/2}$

$= \ln[x(x+2)^{1/2}]$

$= \ln x + \ln(x+2)^{1/2}$

$= \ln x + \frac{1}{2} \ln(x+2)$

61. $\ln x + \ln 3 = \ln 3x$

62. $\ln y + \ln t = \ln yt = \ln ty$

63. $\log_4 z - \log_4 y = \log_4 \dfrac{z}{y}$

64. $\log_5 8 - \log_5 t = \log_5 \dfrac{8}{t}$

65. $2 \log_2(x + 4) = \log_2(x + 4)^2$

66. $\dfrac{2}{3} \log_7(z - 2) = \log_7(z - 2)^{2/3}$

67. $\dfrac{1}{4} \log_3 5x = \log_3(5x)^{1/4} = \log_3 \sqrt[4]{5x}$

68. $-4 \log_6 2x = \log_6(2x)^{-4} = \log_6 \dfrac{1}{16x^4}$

69.
$$\begin{aligned}\ln x - 3\ln(x + 1) &= \ln x - \ln(x + 1)^3\\ &= \ln \frac{x}{(x + 1)^3}\end{aligned}$$

70.
$$\begin{aligned}2 \ln 8 + 5 \ln(z - 4) &= \ln 8^2 + \ln(z - 4)^5\\ &= \ln 64 + \ln(z - 4)^5\\ &= \ln 64(z - 4)^5\end{aligned}$$

71.
$$\begin{aligned}\log x - 2 \log y + 3 \log z &= \log x - \log y^2 + \log z^3\\ &= \log \frac{x}{y^2} + \log z^3 = \log \frac{xz^3}{y^2}\end{aligned}$$

72.
$$\begin{aligned}3 \log_3 x + 4 \log_3 y - 4 \log_3 z &= \log_3 x^3 + \log_3 y^4 - \log_3 z^4\\ &= \log_3 x^3y^4 - \log_3 z^4\\ &= \log_3 \frac{x^3y^4}{z^4}\end{aligned}$$

73.
$$\begin{aligned}\ln x - 4[\ln(x + 2) + \ln(x - 2)] &= \ln x - 4 \ln(x + 2)(x - 2)\\ &= \ln x - 4 \ln(x^2 - 4)\\ &= \ln x - \ln(x^2 - 4)^4\\ &= \ln \frac{x}{(x^2 - 4)^4}\end{aligned}$$

74.
$$\begin{aligned}4[\ln z + \ln(z + 5)] - 2 \ln(z - 5) &= 4[\ln z(z + 5)] - \ln(z - 5)^2\\ &= \ln[z(z + 5)]^4 - \ln(z - 5)^2\\ &= \ln \frac{z^4(z + 5)^4}{(z - 5)^2}\end{aligned}$$

75.
$$\begin{aligned}\frac{1}{3}[2 \ln(x + 3) + \ln x - \ln(x^2 - 1)] &= \frac{1}{3}[\ln(x + 3)^2 + \ln x - \ln(x^2 - 1)]\\ &= \frac{1}{3}[\ln x(x + 3)^2 - \ln(x^2 - 1)]\\ &= \frac{1}{3} \ln \frac{x(x + 3)^2}{x^2 - 1}\\ &= \ln \sqrt[3]{\frac{x(x + 3)^2}{x^2 - 1}}\end{aligned}$$

76.
$$\begin{aligned}2[3 \ln x - \ln(x + 1) - \ln(x - 1)] &= 2[\ln x^3 - \ln(x + 1) - \ln(x - 1)]\\ &= 2[\ln x^3 - [\ln(x + 1) + \ln(x - 1)]]\\ &= 2[\ln x^3 - [\ln(x + 1)(x - 1)]]\\ &= 2 \ln \frac{x^3}{x^2 - 1}\\ &= \ln\left(\frac{x^3}{x^2 - 1}\right)^2\end{aligned}$$

77. $\frac{1}{3}[\log_8 y + 2\log_8(y+4)] - \log_8(y-1) = \frac{1}{3}[\log_8 y + \log_8(y+4)^2] - \log_8(y-1)$

$$= \frac{1}{3}\log_8 y(y+4)^2 - \log_8(y-1)$$
$$= \log_8 \sqrt[3]{y(y+4)^2} - \log_8(y-1)$$
$$= \log_8\left(\frac{\sqrt[3]{y(y+4)^2}}{y-1}\right)$$

78. $\frac{1}{2}[\log_4(x+1) + 2\log_4(x-1)] + 6\log_4 x = \frac{1}{2}[\log_4(x+1) + \log_4(x-1)^2] + \log_4 x^6$

$$= \tfrac{1}{2}[\log_4(x+1)(x-1)^2] + \log_4 x^6$$
$$= \log_4\left[\sqrt{x+1}(x-1)\right] + \log_4 x^6$$
$$= \log_4\left[x^6(x-1)\sqrt{x+1}\right]$$

79. $\log_2 \frac{32}{4} = \log_2 32 - \log_2 4 \neq \frac{\log_2 32}{\log_2 4}$

The second and third expressions are equal by Property 2.

80. $\log_7\sqrt{70} = \frac{1}{2}\log_7 70 = \frac{1}{2}[\log_7 7 + \log_7 10]$

$$= \frac{1}{2}[1 + \log_7 10]$$
$$= \frac{1}{2} + \frac{1}{2}\log_7 10$$

$= \frac{1}{2} + \log_7 \sqrt{10}$ by Property 1 and Property 3

81. $\beta = 10\log\left(\frac{I}{10^{-12}}\right)$

$$= 10[\log I - \log 10^{-12}]$$
$$= 10[\log I + 12]$$
$$= 120 + 10\log I$$

When $I = 10^{-6}$:

$$\beta = 120 + 10\log 10^{-6}$$
$$= 120 + 10(-6)$$
$$= 60 \text{ decibels}$$

82. $\beta = 10\log\left(\frac{I}{10^{-12}}\right)$

$$\text{Difference} = 10\log\left(\frac{3.16\times 10^{-5}}{10^{-12}}\right) - 10\log\left(\frac{1.26\times 10^{-7}}{10^{-12}}\right)$$
$$= 10(\log(3.16\times 10^{7}) - \log(1.26\times 10^{5}))$$
$$= 10\left(\log\left(\frac{3.16\times 10^{7}}{1.26\times 10^{5}}\right)\right)$$
$$= 10(\log(2.5079\times 10^{2}))$$
$$= 10(\log(250.79))$$
$$= 24 \text{ dB}$$

83. $\beta = 120 + 10\log(2I)$

$$= 120 + 10(\log 2 + \log I)$$
$$= (120 + 10\log I) + 10\log 2$$

With both stereos playing, the music is $10\log 2 \approx 3$ decibels louder.

84. $f(t) = 90 - 15\log(t+1),\ 0 \le t \le 12$

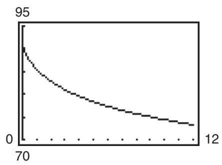
(a) $f(t) = 90 - \log(t+1)^{15}$

(b) $f(0) = 90$

(c) $f(4) = 90 - 15 \cdot \log(4+1) = 79.5$

(d) $f(12) = 90 - 15 \cdot \log(12+1) = 73.3$

(e)

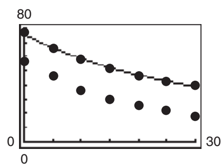


(f) The average score will be 75 when $t = 9$ months. See graph in (e).

(g)
$$75 = 90 - 15\log(t+1)$$
$$-15 = -15\log(t+1)$$
$$1 = \log(t+1)$$
$$10^1 = t+1$$
$$t = 9 \text{ months}$$

85. By using the regression feature on a graphing calculator we obtain $y \approx 256.24 - 20.8\ln x$.

86. (a)



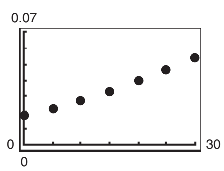
(b) $T - 21 = 54.4(0.964)^t$

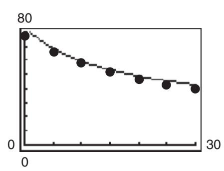
$$T = 54.4(0.964)^t + 21$$

See graph in (a).

(d) $\dfrac{1}{T-21} = 0.0012t + 0.016$

$$T = \frac{1}{0.0012t + 0.016} + 21$$



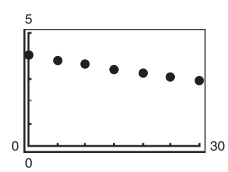


(e) Since the scatter plot of the original data is so nicely exponential, there is no need to do the transformations unless one desires to deal with smaller numbers. The transformations did not make the problem simpler.

Taking logs of temperatures led to a linear scatter plot because the log function increases very slowly as the x-values increase. Taking the reciprocals of the temperatures led to a linear scatter plot because of the asymptotic nature of the reciprocal function.

(c)

t (in minutes)	T (°C)	$T - 21$ (°C)	$\ln(T - 21)$	$1/(T - 21)$
0	78	57	4.043	0.0175
5	66	45	3.807	0.0222
10	57.5	36.5	3.597	0.0274
15	51.2	30.2	3.408	0.0331
20	46.3	25.3	3.231	0.0395
25	42.5	21.5	3.068	0.0465
30	39.6	18.6	2.923	0.0538



$$\ln(T - 21) = -0.037t + 4$$
$$T = e^{-0.037t+4} + 21$$

This graph is identical to T in (b).

87. $f(x) = \ln x$

False, $f(0) \ne 0$ since 0 is not in the domain of $f(x)$.

$f(1) = \ln 1 = 0$

88. $f(ax) = f(a) + f(x),\ a > 0,\ x > 0$

True, because $f(ax) = \ln ax = \ln a + \ln x = f(a) + f(x)$.

89. False. $f(x) - f(2) = \ln x - \ln 2 = \ln \dfrac{x}{2} \neq \ln(x - 2)$

90. $\sqrt{f(x)} = \dfrac{1}{2}f(x)$; false

$\sqrt{f(x)} = \sqrt{\ln x}$ can't be simplified further.

$f\left(\sqrt{x}\right) = \ln \sqrt{x} = \ln x^{1/2} = \dfrac{1}{2} \ln x = \dfrac{1}{2}f(x)$

91. False.

$f(u) = 2f(v) \implies \ln u = 2 \ln v \implies \ln u = \ln v^2 \implies u = v^2$

92. If $f(x) < 0$, then $0 < x < 1$.

True

93. Let $x = \log_b u$ and $y = \log_b v$, then $b^x = u$ and $b^y = v$.

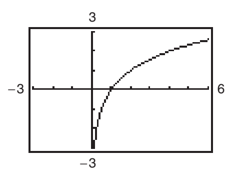
$\dfrac{u}{v} = \dfrac{b^x}{b^y} = b^{x-y}$

Then $\log_b(u/v) = \log_b(b^{x-y}) = x - y = \log_b u - \log_b v$.

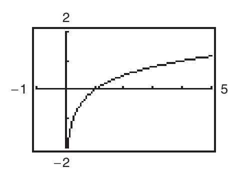
94. Let $x = \log_b u$, then $u = b^x$ and $u^n = b^{nx}$.

$\log_b u^n = \log_b b^{nx} = nx = n \log_b u$

95. $f(x) = \log_2 x = \dfrac{\log x}{\log 2} = \dfrac{\ln x}{\ln 2}$

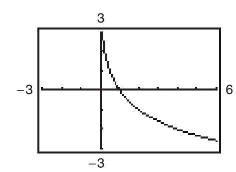


96. $f(x) = \log_4 x = \dfrac{\log x}{\log 4} = \dfrac{\ln x}{\ln 4}$



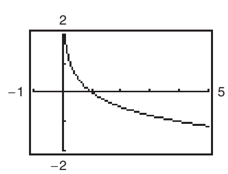
97. $f(x) = \log_{1/2} x$

$= \dfrac{\log x}{\log(1/2)} = \dfrac{\ln x}{\ln(1/2)}$



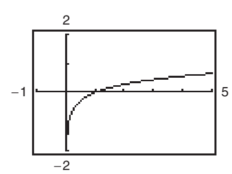
98. $f(x) = \log_{1/4} x$

$= \dfrac{\log x}{\log(1/4)} = \dfrac{\ln x}{\ln(1/4)}$



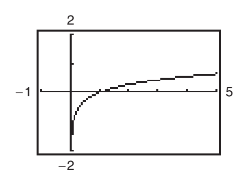
99. $f(x) = \log_{11.8} x$

$= \dfrac{\log x}{\log 11.8} = \dfrac{\ln x}{\ln 11.8}$



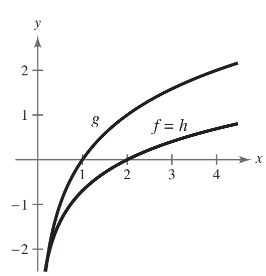
100. $f(x) = \log_{12.4} x$

$= \dfrac{\log x}{\log 12.4} = \dfrac{\ln x}{\ln 12.4}$



101. $f(x) = \ln \dfrac{x}{2}$, $g(x) = \dfrac{\ln x}{\ln 2}$, $h(x) = \ln x - \ln 2$

$f(x) = h(x)$ by Property 2



102. $\ln 2 \approx 0.6931$, $\ln 3 \approx 1.0986$, $\ln 5 \approx 1.6094$

$\ln 2 \approx 0.6931$

$\ln 3 \approx 1.0986$

$\ln 4 = \ln(2 \cdot 2) = \ln 2 + \ln 2 \approx 0.6931 + 0.6931 = 1.3862$

$\ln 5 \approx 1.6094$

$\ln 6 = \ln(2 \cdot 3) = \ln 2 + \ln 3 \approx 0.6931 + 1.0986 = 1.7917$

$\ln 8 = \ln 2^3 = 3 \ln 2 \approx 3(0.6931) = 2.0793$

$\ln 9 = \ln 3^2 = 2 \ln 3 \approx 2(1.0986) = 2.1972$

$\ln 10 = \ln(5 \cdot 2) = \ln 5 + \ln 2 \approx 1.6094 + 0.6931 = 2.3025$

$\ln 12 = \ln(2^2 \cdot 3) = \ln 2^2 + \ln 3 = 2 \ln 2 + \ln 3 \approx 2(0.6931) + 1.0986 = 2.4848$

$\ln 15 = \ln(5 \cdot 3) = \ln 5 + \ln 3 \approx 1.6094 + 1.0986 = 2.7080$

$\ln 16 = \ln 2^4 = 4 \ln 2 \approx 4(0.6931) = 2.7724$

$\ln 18 = \ln(3^2 \cdot 2) = \ln 3^2 + \ln 2 = 2 \ln 3 + \ln 2 \approx 2(1.0986) + 0.6931 = 2.8903$

$\ln 20 = \ln(5 \cdot 2^2) = \ln 5 + \ln 2^2 = \ln 5 + 2 \ln 2 \approx 1.6094 + 2(0.6931) = 2.9956$

103. $\dfrac{24xy^{-2}}{16x^{-3}y} = \dfrac{24xx^3}{16yy^2} = \dfrac{3x^4}{2y^3}, x \neq 0$

104. $\left(\dfrac{2x^2}{3y}\right)^{-3} = \left(\dfrac{3y}{2x^2}\right)^3 = \dfrac{(3y)^3}{(2x^2)^3} = \dfrac{27y^3}{8x^6}$

105. $(18x^3y^4)^{-3}(18x^3y^4)^3 = \dfrac{(18x^3y^4)^3}{(18x^3y^4)^3} = 1$ if $x \neq 0, y \neq 0$.

106.
$$\begin{aligned} xy(x^{-1} + y^{-1})^{-1} &= \frac{xy}{x^{-1} + y^{-1}} \\ &= \frac{xy}{(1/x) + (1/y)} \\ &= \frac{xy}{(y + x)/xy} = \frac{(xy)^2}{x + y} \end{aligned}$$

107.
$$\begin{aligned} 3x^2 + 2x - 1 &= 0 \\ (3x - 1)(x + 1) &= 0 \\ 3x - 1 = 0 &\implies x = \tfrac{1}{3} \\ x + 1 = 0 &\implies x = -1 \end{aligned}$$

108.
$$\begin{aligned} 4x^2 - 5x + 1 &= 0 \\ (4x - 1)(x - 1) &= 0 \\ 4x - 1 = 0 &\implies x = \tfrac{1}{4} \\ x - 1 = 0 &\implies x = 1 \end{aligned}$$

The zeros are $x = \frac{1}{4}, 1$.

109.
$$\begin{aligned} \frac{2}{3x + 1} &= \frac{x}{4} \\ (3x + 1)(x) &= (2)(4) \\ 3x^2 + x - 8 &= 0 \\ x &= \frac{-1 \pm \sqrt{1^2 - 4(3)(-8)}}{2(3)} \\ &= \frac{-1 \pm \sqrt{97}}{6} \end{aligned}$$

110.
$$\begin{aligned} \frac{5}{x - 1} &= \frac{2x}{3} \\ 5(3) &= 2x(x - 1) \\ 15 &= 2x^2 - 2x \\ 0 &= 2x^2 - 2x - 15 \\ \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-15)}}{2(2)} &= x \\ \frac{2 \pm \sqrt{124}}{4} &= x \\ \frac{1 \pm \sqrt{31}}{2} &= x \end{aligned}$$

The zeros are $\dfrac{1 \pm \sqrt{31}}{2}$.

Section 3.4 Exponential and Logarithmic Equations

- To solve an exponential equation, isolate the exponential expression, then take the logarithm of both sides. Then solve for the variable.
  1. $\log_a a^x = x$ 2. $\ln e^x = x$
- To solve a logarithmic equation, rewrite it in exponential form. Then solve for the variable.
  1. $a^{\log_a x} = x$ 2. $e^{\ln x} = x$
- If $a > 0$ and $a \neq 1$ we have the following:
  1. $\log_a x = \log_a y \iff x = y$
  2. $a^x = a^y \iff x = y$
- Check for extraneous solutions.

Vocabulary Check

1. solve

2. (a) $x = y$ (b) $x = y$ (c) x (d) x

3. extraneous

1. $4^{2x-7} = 64$

(a) $x = 5$

$4^{2(5)-7} = 4^3 = 64$

Yes, $x = 5$ *is* a solution.

(b) $x = 2$

$4^{2(2)-7} = 4^{-3} = \frac{1}{64} \neq 64$

No, $x = 2$ *is not* a solution.

2. $2^{3x+1} = 32$

(a) $x = -1$

$2^{3(-1)+1} = 2^{-2} = \frac{1}{4}$

No, $x = -1$ *is not* a solution.

(b) $x = 2$

$2^{3(2)+1} = 2^7 = 128$

No, $x = 2$ *is not* a solution.

3. $3e^{x+2} = 75$

(a) $x = -2 + e^{25}$

$3e^{(-2+e^{25})+2} = 3e^{e^{25}} \neq 75$

No, $x = -2 + e^{25}$ *is not* a solution.

(b) $x = -2 + \ln 25$

$3e^{(-2+\ln 25)+2} = 3e^{\ln 25} = 3(25) = 75$

Yes, $x = -2 + \ln 25$ *is* a solution.

(c) $x \approx 1.219$

$3e^{1.219+2} = 3e^{3.219} \approx 75$

Yes, $x \approx 1.219$ *is* a solution.

4. $2e^{5x+2} = 12$

(a) $x = \frac{1}{5}(-2 + \ln 6)$

$$2e^{5[(1/5)(-2+\ln 6)]+2} = 2e^{-2+\ln 6+2}$$
$$= 2e^{\ln 6} = 2 \cdot 6 = 12$$

Yes, $x = \frac{1}{5}(-2 + \ln 6)$ *is* a solution.

(b) $x = \frac{\ln 6}{5 \ln 2}$

$$2e^{5[\ln 6/(5\ln 2)]+2} = 2e^{(\ln 6/\ln 2)+2}$$
$$\approx 2e^{2.585+2}$$
$$\approx 2 \cdot 97.9995 = 195.999$$

No, $x = \frac{\ln 6}{5 \ln 2}$ *is not* a solution.

(c) $x = -0.0416$

$2e^{5(-0.0416)+2} = 2e^{1.792} \approx 2(6.00144) \approx 12$

Yes, $x = -0.0416$ *is* an approximate solution.

5. $\log_4(3x) = 3 \implies 3x = 4^3 \implies 3x = 64$

(a) $x \approx 21.333$

$3(21.333) \approx 64$

Yes, 21.333 *is* an approximate solution.

(b) $x = -4$

$3(-4) = -12 \neq 64$

No, $x = -4$ *is not* a solution.

(c) $x = \frac{64}{3}$

$3\left(\frac{64}{3}\right) = 64$

Yes, $x = \frac{64}{3}$ *is* a solution.

6. $\log_2(x + 3) = 10$

(a) $x = 1021$

$\log_2(1021 + 3) = \log_2(1024)$

Since $2^{10} = 1024$, $x = 1021$ *is* a solution.

(b) $x = 17$

$\log_2(17 + 3) = \log_2(20)$

Since $2^{10} \neq 20$, $x = 17$ *is not* a solution.

(c) $x = 10^2 - 3 = 97$

$\log_2(97 + 3) = \log_2(100)$

Since $2^{10} \neq 100$, $10^2 - 3$ *is not* a solution.

7. $\ln(2x + 3) = 5.8$

(a) $x = \frac{1}{2}(-3 + \ln 5.8)$

$\ln\left[2\left(\frac{1}{2}\right)(-3 + \ln 5.8) + 3\right] = \ln(\ln 5.8) \neq 5.8$

No, $x = \frac{1}{2}(-3 + \ln 5.8)$ *is not* a solution.

(b) $x = \frac{1}{2}(-3 + e^{5.8})$

$\ln\left[2\left(\frac{1}{2}\right)(-3 + e^{5.8}) + 3\right] = \ln(e^{5.8}) = 5.8$

Yes, $x = \frac{1}{2}(-3 + e^{5.8})$ *is* a solution.

(c) $x \approx 163.650$

$\ln[2(163.650) + 3] = \ln 330.3 \approx 5.8$

Yes, $x \approx 163.650$ *is* an approximate solution.

8. $\ln(x - 1) = 3.8$

(a) $x = 1 + e^{3.8}$

$\ln(1 + e^{3.8} - 1) = \ln e^{3.8} = 3.8$

Yes, $x = 1 + e^{3.8}$ *is* a solution.

(b) $x \approx 45.701$

$\ln(45.701 - 1) = \ln(44.701) \approx 3.8$

Yes, $x \approx 45.701$ *is* an approximate solution.

(c) $x = 1 + \ln 3.8$

$\ln(1 + \ln 3.8 - 1) = \ln(\ln 3.8) \approx 0.289$

No, $x = 1 + \ln 3.8$ *is not* a solution.

9. $4^x = 16$

$4^x = 4^2$

$x = 2$

10. $3^x = 243$

$3^x = 3^5$

$x = 5$

11. $\left(\frac{1}{2}\right)^x = 32$

$2^{-x} = 2^5$

$-x = 5$

$x = -5$

12. $\left(\frac{1}{4}\right)^x = 64$

$4^{-x} = 4^3$

$-x = 3$

$x = -3$

13. $\ln x - \ln 2 = 0$

$\ln x = \ln 2$

$x = 2$

14. $\ln x - \ln 5 = 0$

$\ln x = \ln 5$

$x = 5$

15. $e^x = 2$

$\ln e^x = \ln 2$

$x = \ln 2$

$x \approx 0.693$

16. $e^x = 4$

$\ln e^x = \ln 4$

$x = \ln 4$

$x \approx 1.386$

17. $\ln x = -1$

$e^{\ln x} = e^{-1}$

$x = e^{-1}$

$x \approx 0.368$

18. $\ln x = -7$

$e^{\ln x} = e^{-7}$

$x = e^{-7}$

$x \approx 0.000912$

19. $\log_4 x = 3$

$4^{\log_4 x} = 4^3$

$x = 4^3$

$x = 64$

20. $\log_5 x = -3$

$x = 5^{-3}$

$x = \frac{1}{125}$ or 0.008

21. $f(x) = g(x)$

$2^x = 8$

$2^x = 2^3$

$x = 3$

Point of intersection: $(3, 8)$

22. $f(x) = g(x)$

$27^x = 9$

$27^x = 27^{2/3}$

$x = \frac{2}{3}$

Point of intersection: $\left(\frac{2}{3}, 9\right)$

23. $f(x) = g(x)$

$\log_3 x = 2$

$x = 3^2$

$x = 9$

Point of intersection: $(9, 2)$

24. $f(x) = g(x)$

$\ln(x - 4) = 0$

$e^{\ln(x-4)} = e^0$

$x - 4 = 1$

$x = 5$

Point of intersection: $(5, 0)$

25. $e^x = e^{x^2-2}$

$x = x^2 - 2$

$0 = x^2 - x - 2$

$0 = (x + 1)(x - 2)$

$x = -1$ or $x = 2$

26. $e^{2x} = e^{x^2-8}$

$2x = x^2 - 8$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = -2,\ x = 4$

27. $e^{x^2-3} = e^{x-2}$

$x^2 - 3 = x - 2$

$x^2 - x - 1 = 0$

By the Quadratic Formula $x \approx 1.618$ or $x \approx -0.618$.

28. $e^{-x^2} = e^{x^2-2x}$

$-x^2 = x^2 - 2x$

$2x^2 - 2x = 0$

$2x(x - 1) = 0$

$x = 0,\ x = 1$

29. $4(3^x) = 20$

$3^x = 5$

$\log_3 3^x = \log_3 5$

$x = \log_3 5 = \dfrac{\log 5}{\log 3}$ or $\dfrac{\ln 5}{\ln 3}$

$x \approx 1.465$

30. $2(5^x) = 32$

$5^x = 16$

$x = \log_5 16$

$x = \dfrac{\ln 16}{\ln 5}$

$x \approx 1.723$

31. $2e^x = 10$

$e^x = 5$

$\ln e^x = \ln 5$

$x = \ln 5 \approx 1.609$

32. $4e^x = 91$

$e^x = \frac{91}{4}$

$\ln e^x = \ln \frac{91}{4}$

$x = \ln \frac{91}{4} \approx 3.125$

33. $e^x - 9 = 19$

$e^x = 28$

$\ln e^x = \ln 28$

$x = \ln 28 \approx 3.332$

34. $6^x + 10 = 47$

$6^x = 37$

$x = \log_6 37$

$x = \dfrac{\ln 37}{\ln 6}$

$x \approx 2.015$

35. $3^{2x} = 80$

$\ln 3^{2x} = \ln 80$

$2x \ln 3 = \ln 80$

$x = \dfrac{\ln 80}{2 \ln 3} \approx 1.994$

36. $6^{5x} = 3000$

$\ln 6^{5x} = \ln 3000$

$(5x) \ln 6 = \ln 3000$

$5x = \dfrac{\ln 3000}{\ln 6}$

$x = \dfrac{\ln 3000}{5 \ln 6} \approx 0.894$

37. $5^{-t/2} = 0.20$

$5^{-t/2} = \dfrac{1}{5}$

$5^{-t/2} = 5^{-1}$

$-\dfrac{t}{2} = -1$

$t = 2$

38. $4^{-3t} = 0.10$

$\ln 4^{-3t} = \ln 0.10$

$(-3t) \ln 4 = \ln 0.10$

$-3t = \dfrac{\ln 0.10}{\ln 4}$

$t = -\dfrac{\ln 0.10}{3 \ln 4} \approx 0.554$

39. $3^{x-1} = 27$

$3^{x-1} = 3^3$

$x - 1 = 3$

$x = 4$

40. $2^{x-3} = 32$

$x - 3 = \log_2 32$

$x - 3 = 5$

$x = 8$

41.
$$\begin{aligned} 2^{3-x} &= 565 \\ \ln 2^{3-x} &= \ln 565 \\ (3-x)\ln 2 &= \ln 565 \\ 3\ln 2 - x\ln 2 &= \ln 565 \\ -x\ln 2 &= \ln 565 - 3\ln 2 \\ x\ln 2 &= 3\ln 2 - \ln 565 \\ x &= \frac{3\ln 2 - \ln 565}{\ln 2} \\ &= 3 - \frac{\ln 565}{\ln 2} \approx -6.142 \end{aligned}$$

42.
$$\begin{aligned} 8^{-2-x} &= 431 \\ \ln 8^{-2-x} &= \ln 431 \\ (-2-x)\ln 8 &= \ln 431 \\ -2\ln 8 - x\ln 8 &= \ln 431 \\ -x\ln 8 &= \ln 431 + \ln 8^2 \\ x\ln 8 &= -\ln 431 - \ln 64 \\ x &= \frac{-\ln 431 - \ln 64}{\ln 8} \approx -4.917 \end{aligned}$$

43.
$$\begin{aligned} 8(10^{3x}) &= 12 \\ 10^{3x} &= \frac{12}{8} \\ \log 10^{3x} &= \log\left(\frac{3}{2}\right) \\ 3x &= \log\left(\frac{3}{2}\right) \\ x &= \frac{1}{3}\log\left(\frac{3}{2}\right) \\ &\approx 0.059 \end{aligned}$$

44.
$$\begin{aligned} 5(10^{x-6}) &= 7 \\ 10^{x-6} &= \frac{7}{5} \\ \log 10^{x-6} &= \log\frac{7}{5} \\ x-6 &= \log\frac{7}{5} \\ x &= 6 + \log\frac{7}{5} \\ &\approx 6.146 \end{aligned}$$

45.
$$\begin{aligned} 3(5^{x-1}) &= 21 \\ 5^{x-1} &= 7 \\ \ln 5^{x-1} &= \ln 7 \\ (x-1)\ln 5 &= \ln 7 \\ x-1 &= \frac{\ln 7}{\ln 5} \\ x &= 1 + \frac{\ln 7}{\ln 5} \approx 2.209 \end{aligned}$$

46.
$$\begin{aligned} 8(3^{6-x}) &= 40 \\ 3^{6-x} &= 5 \\ \ln 3^{6-x} &= \ln 5 \\ (6-x)\ln 3 &= \ln 5 \\ 6-x &= \frac{\ln 5}{\ln 3} \\ -x &= \frac{\ln 5}{\ln 3} - 6 \\ x &= 6 - \frac{\ln 5}{\ln 3} \approx 4.535 \end{aligned}$$

47.
$$\begin{aligned} e^{3x} &= 12 \\ 3x &= \ln 12 \\ x &= \frac{\ln 12}{3} \approx 0.828 \end{aligned}$$

48.
$$\begin{aligned} e^{2x} &= 50 \\ \ln e^{2x} &= \ln 50 \\ 2x &= \ln 50 \\ x &= \frac{\ln 50}{2} \approx 1.956 \end{aligned}$$

49.
$$\begin{aligned} 500e^{-x} &= 300 \\ e^{-x} &= \tfrac{3}{5} \\ -x &= \ln\tfrac{3}{5} \\ x &= -\ln\tfrac{3}{5} \\ &= \ln\tfrac{5}{3} \approx 0.511 \end{aligned}$$

50.
$$\begin{aligned} 1000e^{-4x} &= 75 \\ e^{-4x} &= \tfrac{3}{40} \\ \ln e^{-4x} &= \ln\tfrac{3}{40} \\ -4x &= \ln\tfrac{3}{40} \\ x &= -\tfrac{1}{4}\ln\tfrac{3}{40} \\ &\approx 0.648 \end{aligned}$$

51.
$$\begin{aligned} 7 - 2e^x &= 5 \\ -2e^x &= -2 \\ e^x &= 1 \\ x &= \ln 1 = 0 \end{aligned}$$

52.
$$\begin{aligned} -14 + 3e^x &= 11 \\ 3e^x &= 25 \\ e^x &= \tfrac{25}{3} \\ \ln e^x &= \ln\tfrac{25}{3} \\ x &= \ln\tfrac{25}{3} \\ &\approx 2.120 \end{aligned}$$

53. $$6(2^{3x-1}) - 7 = 9$$
$$6(2^{3x-1}) = 16$$
$$2^{3x-1} = \frac{8}{3}$$
$$\log_2 2^{3x-1} = \log_2\left(\frac{8}{3}\right)$$
$$3x - 1 = \log_2\left(\frac{8}{3}\right) = \frac{\log(8/3)}{\log 2} \text{ or } \frac{\ln(8/3)}{\ln 2}$$
$$x = \frac{1}{3}\left[\frac{\log(8/3)}{\log 2} + 1\right] \approx 0.805$$

54. $$8(4^{6-2x}) + 13 = 41$$
$$8(4^{6-2x}) = 28$$
$$4^{6-2x} = 3.5$$
$$6 - 2x = \log_4 3.5$$
$$6 - 2x = \frac{\ln 3.5}{\ln 4}$$
$$-2x = -6 + \frac{\ln 3.5}{\ln 4}$$
$$x = 3 - \frac{\ln 3.5}{2 \ln 4} \approx 2.548$$

55. $$e^{2x} - 4e^x - 5 = 0$$
$$(e^x + 1)(e^x - 5) = 0$$
$e^x = -1$ or $e^x = 5$

(No solution) $x = \ln 5 \approx 1.609$

56. $$e^{2x} - 5e^x + 6 = 0$$
$$(e^x - 2)(e^x - 3) = 0$$
$$e^x = 2 \text{ or } e^x = 3$$
$$x = \ln 2 \approx 0.693 \text{ or } x = \ln 3 \approx 1.099$$

57. $$e^{2x} - 3e^x - 4 = 0$$
$$(e^x + 1)(e^x - 4) = 0$$
$$e^x + 1 = 0 \implies e^x = -1$$
Not possible since $e^x > 0$ for all x.
$$e^x - 4 = 0 \implies e^x = 4 \implies x = \ln 4 \approx 1.386$$

58. $$e^{2x} + 9e^x + 36 = 0$$
$$(e^x)^2 + 9e^x + 36 = 0$$
Because the discriminant is $9^2 - 4(1)(36) = -63$, there is no solution.

59. $$\frac{500}{100 - e^{x/2}} = 20$$
$$500 = 20(100 - e^{x/2})$$
$$25 = 100 - e^{x/2}$$
$$e^{x/2} = 75$$
$$\frac{x}{2} = \ln 75$$
$$x = 2 \ln 75 \approx 8.635$$

60. $$\frac{400}{1 + e^{-x}} = 350$$
$$400 = 350(1 + e^{-x})$$
$$\frac{8}{7} = 1 + e^{-x}$$
$$\frac{8}{7} - 1 = e^{-x}$$
$$\frac{1}{7} = e^{-x}$$
$$\ln \frac{1}{7} = \ln e^{-x}$$
$$-x = \ln \frac{1}{7}$$
$$-x = \ln 7^{-1}$$
$$-x = -\ln 7$$
$$x = \ln 7 \approx 1.946$$

61. $$\frac{3000}{2 + e^{2x}} = 2$$
$$3000 = 2(2 + e^{2x})$$
$$1500 = 2 + e^{2x}$$
$$1498 = e^{2x}$$
$$\ln 1498 = 2x$$
$$x = \frac{\ln 1498}{2} \approx 3.656$$

62. $\dfrac{119}{e^{6x} - 14} = 7$

$$119 = 7(e^{6x} - 14)$$
$$17 = e^{6x} - 14$$
$$31 = e^{6x}$$
$$\ln 31 = \ln e^{6x}$$
$$\ln 31 = 6x$$
$$x = \frac{\ln 31}{6} \approx 0.572$$

63. $\left(1 + \dfrac{0.065}{365}\right)^{365t} = 4$

$$\ln\left(1 + \frac{0.065}{365}\right)^{365t} = \ln 4$$
$$365t \ln\left(1 + \frac{0.065}{365}\right) = \ln 4$$
$$t = \frac{\ln 4}{365 \ln\left(1 + \frac{0.065}{365}\right)} \approx 21.330$$

64. $\left(4 - \dfrac{2.471}{40}\right)^{9t} = 21$

$$3.938225^{9t} = 21$$
$$\ln 3.938225^{9t} = \ln 21$$
$$9t \ln 3.938225 = \ln 21$$
$$t = \frac{\ln 21}{9 \ln 3.938225} \approx 0.247$$

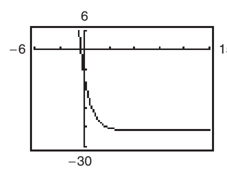
65. $\left(1 + \dfrac{0.10}{12}\right)^{12t} = 2$

$$\ln\left(1 + \frac{0.10}{12}\right)^{12t} = \ln 2$$
$$12t \ln\left(1 + \frac{0.10}{12}\right) = \ln 2$$
$$t = \frac{\ln 2}{12 \ln\left(1 + \frac{0.10}{12}\right)} \approx 6.960$$

66. $\left(16 - \dfrac{0.878}{26}\right)^{3t} = 30$

$$\ln\left(16 - \frac{0.878}{26}\right)^{3t} = \ln 30$$
$$3t \ln\left(16 - \frac{0.878}{26}\right) = \ln 30$$
$$t = \frac{\ln 30}{3 \ln\left(16 - \frac{0.878}{26}\right)} \approx 0.409$$

67. $g(x) = 6e^{1-x} - 25$

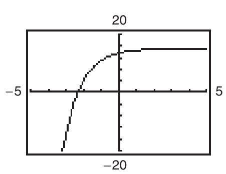


Algebraically:

$$6e^{1-x} = 25$$
$$e^{1-x} = \frac{25}{6}$$
$$1 - x = \ln\left(\frac{25}{6}\right)$$
$$x = 1 - \ln\left(\frac{25}{6}\right)$$
$$x \approx -0.427$$

The zero is $x \approx -0.427$.

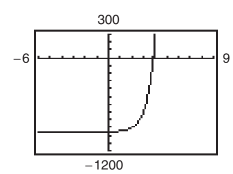
68. $f(x) = -4e^{-x-1} + 15$



$$0 = -4e^{-x-1} + 15$$
$$-15 = -4e^{-x-1}$$
$$3.75 = e^{-x-1}$$
$$\ln 3.75 = -x - 1$$
$$1 + \ln 3.75 = -x$$
$$-1 - \ln 3.75 = x$$
$$-2.322 \approx x$$

The zero is -2.322.

69. $f(x) = 3e^{3x/2} - 962$



Algebraically:

$$3e^{3x/2} = 962$$
$$e^{3x/2} = \frac{962}{3}$$
$$\frac{3x}{2} = \ln\left(\frac{962}{3}\right)$$
$$x = \frac{2}{3}\ln\left(\frac{962}{3}\right)$$
$$x \approx 3.847$$

The zero is $x \approx 3.847$.

70. $g(x) = 8e^{-2x/3} - 11$

$8e^{-2x/3} = 11$

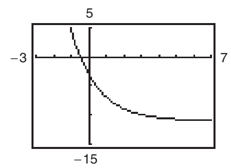
$e^{-2x/3} = 1.375$

$-\dfrac{2x}{3} = \ln 1.375$

$x = -1.5 \ln 1.375$

$x \approx -0.478$

The zero is -0.478.



71. $g(t) = e^{0.09t} - 3$

Algebraically:

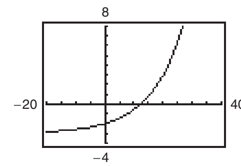
$e^{0.09t} = 3$

$0.09t = \ln 3$

$t = \dfrac{\ln 3}{0.09}$

$t \approx 12.207$

The zero is $t \approx 12.207$.



72. $f(x) = -e^{1.8x} + 7$

$-e^{1.8x} + 7 = 0$

$-e^{1.8x} = -7$

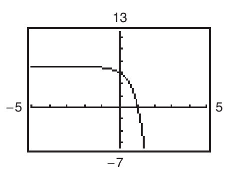
$e^{1.8x} = 7$

$1.8x = \ln 7$

$x = \dfrac{\ln 7}{1.8}$

$x \approx 1.081$

The zero is 1.081.



73. $h(t) = e^{0.125t} - 8$

Algebraically:

$e^{0.125t} - 8 = 0$

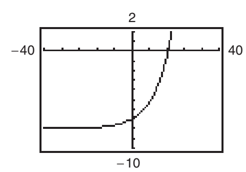
$e^{0.125t} = 8$

$0.125t = \ln 8$

$t = \dfrac{\ln 8}{0.125}$

$t \approx 16.636$

The zero is $t \approx 16.636$.



74. $f(x) = e^{2.724x} - 29$

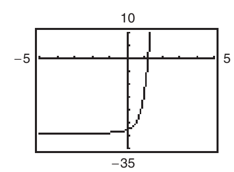
$e^{2.724x} = 29$

$2.724x = \ln 29$

$x = \dfrac{\ln 29}{2.724}$

$x \approx 1.236$

The zero is 1.236.



75. $\ln x = -3$

$x = e^{-3} \approx 0.050$

76. $\ln x = 2$

$e^{\ln x} = e^2$

$x = e^2 \approx 7.389$

77. $\ln 2x = 2.4$

$2x = e^{2.4}$

$x = \dfrac{e^{2.4}}{2} \approx 5.512$

78. $\ln 4x = 1$

$e^{\ln 4x} = e^1$

$4x = e$

$x = \dfrac{e}{4} \approx 0.680$

79. $\log x = 6$

$x = 10^6$

$= 1{,}000{,}000.000$

80. $\log 3z = 2$

$10^{\log 3z} = 10^2$

$3z = 100$

$z = \dfrac{100}{3} \approx 33.333$

81. $3 \ln 5x = 10$

$\ln 5x = \dfrac{10}{3}$

$5x = e^{10/3}$

$x = \dfrac{e^{10/3}}{5} \approx 5.606$

82. $2 \ln x = 7$

$\ln x = \dfrac{7}{2}$

$e^{\ln x} = e^{7/2}$

$x = e^{7/2} \approx 33.115$

83. $\ln \sqrt{x+2} = 1$

$\sqrt{x+2} = e^1$

$x + 2 = e^2$

$x = e^2 - 2$

≈ 5.389

84. $\ln \sqrt{x-8} = 5$

$e^{\ln \sqrt{x-8}} = e^5$

$\sqrt{x-8} = e^5$

$x - 8 = e^{10}$

$x = e^{10} + 8$

$\approx 22{,}034.466$

85. $7 + 3 \ln x = 5$

$3 \ln x = -2$

$\ln x = -\frac{2}{3}$

$x = e^{-2/3}$

≈ 0.513

86. $2 - 6 \ln x = 10$

$-6 \ln x = 8$

$\ln x = -\frac{4}{3}$

$e^{\ln x} = e^{-4/3}$

$x = e^{-4/3}$

≈ 0.264

87. $6\log_3(0.5x) = 11$

$$\log_3(0.5x) = \tfrac{11}{6}$$
$$3^{\log_3(0.5x)} = 3^{11/6}$$
$$0.5x = 3^{11/6}$$
$$x = 2(3^{11/6}) \approx 14.988$$

88. $5\log_{10}(x-2) = 11$

$$\log_{10}(x-2) = \tfrac{11}{5}$$
$$10^{\log_{10}(x-2)} = 10^{11/5}$$
$$x - 2 = 10^{11/5}$$
$$x = 10^{11/5} + 2 \approx 160.489$$

89. $\ln x - \ln(x+1) = 2$

$$\ln\left(\frac{x}{x+1}\right) = 2$$
$$\frac{x}{x+1} = e^2$$
$$x = e^2(x+1)$$
$$x = e^2x + e^2$$
$$x - e^2x = e^2$$
$$x(1-e^2) = e^2$$
$$x = \frac{e^2}{1-e^2} \approx -1.157$$

This negative value is extraneous. The equation has no solution.

90. $\ln x + \ln(x+1) = 1$

$$\ln[x(x+1)] = 1$$
$$e^{\ln[x(x+1)]} = e^1$$
$$x(x+1) = e^1$$
$$x^2 + x - e = 0$$
$$x = \frac{-1 \pm \sqrt{1+4e}}{2}$$

The only solution is $x = \dfrac{-1 + \sqrt{1+4e}}{2} \approx 1.223$.

91. $\ln x + \ln(x-2) = 1$

$$\ln[x(x-2)] = 1$$
$$x(x-2) = e^1$$
$$x^2 - 2x - e = 0$$
$$x = \frac{2 \pm \sqrt{4+4e}}{2}$$
$$= \frac{2 \pm 2\sqrt{1+e}}{2} = 1 \pm \sqrt{1+e}$$

The negative value is extraneous. The only solution is $x = 1 + \sqrt{1+e} \approx 2.928$.

92. $\ln x + \ln(x+3) = 1$

$$\ln[x(x+3)] = 1$$
$$e^{\ln[x(x+3)]} = e^1$$
$$x(x+3) = e^1$$
$$x^2 + 3x - e = 0$$
$$x = \frac{-3 \pm \sqrt{9+4e}}{2}$$

The only solution is $x = \dfrac{-3 + \sqrt{9+4e}}{2} \approx 0.729$.

93. $\ln(x+5) = \ln(x-1) - \ln(x+1)$

$$\ln(x+5) = \ln\left(\frac{x-1}{x+1}\right)$$
$$x + 5 = \frac{x-1}{x+1}$$
$$(x+5)(x+1) = x - 1$$
$$x^2 + 6x + 5 = x - 1$$
$$x^2 + 5x + 6 = 0$$
$$(x+2)(x+3) = 0$$
$$x = -2 \text{ or } x = -3$$

Both of these solutions are extraneous, so the equation has no solution.

94. $\ln(x+1) - \ln(x-2) = \ln x$

$$\ln\left(\frac{x+1}{x-2}\right) = \ln x$$
$$\frac{x+1}{x-2} = x$$
$$x + 1 = x^2 - 2x$$
$$0 = x^2 - 3x - 1$$
$$\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)} = x$$
$$\frac{3 \pm \sqrt{13}}{2} = x$$
$$3.303 \approx x$$

(The negative apparent solution is extraneous.)

95. $\log_2(2x-3) = \log_2(x+4)$

$$2x - 3 = x + 4$$
$$x = 7$$

96. $\log(x - 6) = \log(2x + 1)$

$$x - 6 = 2x + 1$$
$$-7 = x$$

The apparent solution $x = -7$ is extraneous, because the domain of the logarithm function is positive numbers, and $-7 - 6$ and $2(-7) + 1$ are negative. There is no solution.

97. $\log(x + 4) - \log x = \log(x + 2)$

$$\log\left(\frac{x+4}{x}\right) = \log(x + 2)$$
$$\frac{x+4}{x} = x + 2$$
$$x + 4 = x^2 + 2x$$
$$0 = x^2 + x - 4$$
$$x = \frac{-1 \pm \sqrt{17}}{2} \quad \text{Quadratic Formula}$$

Choosing the positive value of x (the negative value is extraneous), we have

$$x = \frac{-1 + \sqrt{17}}{2} \approx 1.562.$$

98. $\log_2 x + \log_2(x + 2) = \log_2(x + 6)$

$$\log_2[x(x + 2)] = \log_2(x + 6)$$
$$x(x + 2) = x + 6$$
$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$x = -3 \text{ or } x = 2$$

The value $x = -3$ is extraneous. The only solution is $x = 2$.

99. $\log_4 x - \log_4(x - 1) = \frac{1}{2}$

$$\log_4\left(\frac{x}{x-1}\right) = \frac{1}{2}$$
$$4^{\log_4[x/(x-1)]} = 4^{1/2}$$
$$\frac{x}{x-1} = 4^{1/2}$$
$$x = 2(x - 1)$$
$$x = 2x - 2$$
$$-x = -2$$
$$x = 2$$

100. $\log_3 x + \log_3(x - 8) = 2$

$$\log_3[x(x - 8)] = 2$$
$$3^{\log_3(x^2 - 8x)} = 3^2$$
$$x^2 - 8x = 9$$
$$x^2 - 8x - 9 = 0$$
$$(x - 9)(x + 1) = 0$$
$$x = 9 \text{ or } x = -1$$

The value $x = -1$ is extraneous. The only solution is $x = 9$.

101. $\log 8x - \log\left(1 + \sqrt{x}\right) = 2$

$$\log \frac{8x}{1 + \sqrt{x}} = 2$$
$$\frac{8x}{1 + \sqrt{x}} = 10^2$$
$$8x = 100\left(1 + \sqrt{x}\right)$$
$$2x = 25\left(1 + \sqrt{x}\right) = 25 + 25\sqrt{x}$$
$$2x - 25 = 25\sqrt{x}$$
$$(2x - 25)^2 = \left(25\sqrt{x}\right)^2$$
$$4x^2 - 100x + 625 = 625x$$
$$4x^2 - 725x + 625 = 0$$
$$x = \frac{725 \pm \sqrt{725^2 - 4(4)(625)}}{2(4)} = \frac{725 \pm \sqrt{515,625}}{8} = \frac{25\left(29 \pm 5\sqrt{33}\right)}{8}$$
$$x \approx 0.866 \text{ (extraneous) or } x \approx 180.384$$

The only solution is $x = \dfrac{25\left(29 + 5\sqrt{33}\right)}{8} \approx 180.384$.

102. $\log 4x - \log(12 + \sqrt{x}) = 2$

$$\log\left(\frac{4x}{12 + \sqrt{x}}\right) = 2$$

$$10^{\log(4x/(12+\sqrt{x}))} = 10^2$$

$$\frac{4x}{12 + \sqrt{x}} = 100$$

$$4x = 100(12 + \sqrt{x})$$

$$4x = 1200 + 100\sqrt{x}$$

$$4x - 1200 = 100\sqrt{x}$$

$$x - 300 = 25\sqrt{x}$$

$$(x - 300)^2 = (25\sqrt{x})^2$$

$$x^2 - 600x + 90{,}000 = 625x$$

$$x^2 - 1225x + 90{,}000 = 0$$

$$x = \frac{1225 \pm \sqrt{(-1225)^2 - 4(1)(90{,}000)}}{2}$$

$$x = \frac{1225 \pm \sqrt{1{,}140{,}625}}{2}$$

$$x = \frac{1225 \pm 125\sqrt{73}}{2}$$

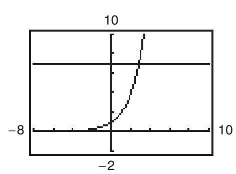
$$x \approx 78.500 \text{ (extraneous) or } x \approx 1146.500$$

The only solution is $x = \dfrac{1225 + 125\sqrt{73}}{2} \approx 1146.500.$

103. $y_1 = 7$

$y_2 = 2^x$

From the graph we have $x \approx 2.807$ when $y = 7$. Algebraically:



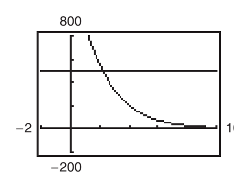
$$2^x = 7$$

$$\ln 2^x = \ln 7$$

$$x \ln 2 = \ln 7$$

$$x = \frac{\ln 7}{\ln 2} \approx 2.807$$

104. $500 = 1500e^{-x/2}$



$$\frac{1}{3} = e^{-x/2}$$

$$\ln\frac{1}{3} = -\frac{x}{2}$$

$$-2\ln\frac{1}{3} = x$$

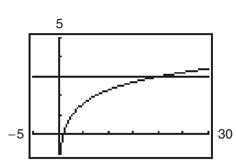
$$2.197 \approx x$$

The solution is $x \approx 2.197$.

105. $y_1 = 3$

$y_2 = \ln x$

From the graph we have $x \approx 20.086$ when $y = 3$. Algebraically:

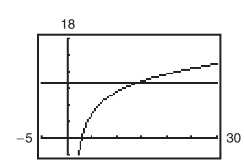


$$3 - \ln x = 0$$

$$\ln x = 3$$

$$x = e^3 \approx 20.086$$

106. $10 - 4\ln(x - 2) = 0$



$$-4\ln(x - 2) = -10$$

$$\ln(x - 2) = 2.5$$

$$e^{\ln(x-2)} = e^{2.5}$$

$$x - 2 = e^{2.5}$$

$$x = e^{2.5} + 2$$

$$x \approx 14.182$$

The solution is $x \approx 14.182$.

107. (a)

$$\begin{aligned} A &= Pe^{rt} \\ 5000 &= 2500e^{0.085t} \\ 2 &= e^{0.085t} \\ \ln 2 &= 0.085t \\ \frac{\ln 2}{0.085} &= t \\ t &\approx 8.2 \text{ years} \end{aligned}$$

(b)

$$\begin{aligned} A &= Pe^{rt} \\ 7500 &= 2500e^{0.085t} \\ 3 &= e^{0.085t} \\ \ln 3 &= 0.085t \\ \frac{\ln 3}{0.085} &= t \\ t &\approx 12.9 \text{ years} \end{aligned}$$

108. (a)

$$\begin{aligned} r &= 0.12 \\ A &= Pe^{rt} \\ 5000 &= 2500e^{0.12t} \\ 2 &= e^{0.12t} \\ \ln 2 &= \ln e^{0.12t} \\ \ln 2 &= 0.12t \\ \frac{\ln 2}{0.12} &= t \\ t &\approx 5.8 \text{ years} \end{aligned}$$

(b)

$$\begin{aligned} r &= 0.12 \\ A &= Pe^{rt} \\ 7500 &= 2500e^{0.12t} \\ 3 &= e^{0.12t} \\ \ln 3 &= \ln e^{0.12t} \\ \ln 3 &= 0.12t \\ \frac{\ln 3}{0.12} &= t \\ t &= 9.2 \text{ years} \end{aligned}$$

109. $p = 500 - 0.5(e^{0.004x})$

(a)

$$\begin{aligned} p &= 350 \\ 350 &= 500 - 0.5(e^{0.004x}) \\ 300 &= e^{0.004x} \\ 0.004x &= \ln 300 \\ x &\approx 1426 \text{ units} \end{aligned}$$

(b)

$$\begin{aligned} p &= 300 \\ 300 &= 500 - 0.5(e^{0.004x}) \\ 400 &= e^{0.004x} \\ 0.004x &= \ln 400 \\ x &\approx 1498 \text{ units} \end{aligned}$$

110. $p = 5000\left(1 - \dfrac{4}{4 + e^{-0.002x}}\right)$

(a) When $p = \$600$:

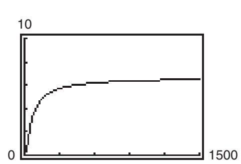
$$\begin{aligned} 600 &= 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right) \\ 0.12 &= 1 - \frac{4}{4 + e^{-0.002x}} \\ \frac{4}{4 + e^{-0.002x}} &= 0.88 \\ 4 &= 3.52 + 0.88e^{-0.002x} \\ 0.48 &= 0.88e^{-0.002x} \\ \frac{6}{11} &= e^{-0.002x} \\ \ln\frac{6}{11} &= \ln e^{-0.002x} \\ \ln\frac{6}{11} &= -0.002x \\ x &= -\frac{\ln(6/11)}{0.002} \approx 303 \text{ units} \end{aligned}$$

(b) When $p = \$400$:

$$\begin{aligned} 400 &= 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right) \\ 0.08 &= 1 - \frac{4}{4 + e^{-0.002x}} \\ \frac{4}{4 + e^{-0.002x}} &= 0.92 \\ 4 &= 3.68 + 0.92e^{-0.002x} \\ 0.32 &= 0.92e^{-0.002x} \\ \frac{8}{23} &= e^{-0.002x} \\ \ln\frac{8}{23} &= \ln e^{-0.002x} \\ \ln\frac{8}{23} &= -0.002x \\ x &= -\frac{\ln(8/23)}{0.002} \approx 528 \text{ units} \end{aligned}$$

111. $V = 6.7e^{-48.1/t},\ t \ge 0$

(a)



(b) As $t \longrightarrow \infty$, $V \longrightarrow 6.7$.

Horizontal asymptote: $V = 6.7$

The yield will approach 6.7 million cubic feet per acre.

(c)

$$\begin{aligned} 1.3 &= 6.7e^{-48.1/t} \\ \frac{1.3}{6.7} &= e^{-48.1/t} \\ \ln\left(\frac{13}{67}\right) &= \frac{-48.1}{t} \\ t &= \frac{-48.1}{\ln(13/67)} \approx 29.3 \text{ years} \end{aligned}$$

112. $N = 68(10^{-0.04x})$

When $N = 21$:

$$21 = 68(10^{-0.04x})$$

$$\frac{21}{68} = 10^{-0.04x}$$

$$\log_{10}\frac{21}{68} = -0.04x$$

$$x = -\frac{\log_{10}(21/68)}{0.04} \approx 12.76 \text{ inches}$$

113. $y = 7312 - 630.0 \ln t,\ 5 \le t \le 12$

$$7312 - 630.0 \ln t = 5800$$

$$-630.0 \ln t = -1512$$

$$\ln t = 2.4$$

$$t = e^{2.4} \approx 11$$

$t \approx 11$ corresponds to the year 2001.

114. $y = 4381 + 1883.6 \ln t,\ 5 \le t \le 13$

$$9000 = 4381 + 1883.6 \ln t$$

$$4619 = 1883.6 \ln t$$

$$\ln t = \frac{4619}{1883.6} = 2.45222$$

$$t = e^{2.45222} = 11.6$$

Since $t = 5$ represents 1995, $t = 11.6$ indicates that the number of daily fee golf facilities in the U.S. reached 9000 in 2001.

115. (a) From the graph shown in the textbook, we see horizontal asymptotes at $y = 0$ and $y = 100$. These represent the lower and upper percent bounds; the range falls between 0% and 100%.

(b) Males

$$50 = \frac{100}{1 + e^{-0.6114(x-69.71)}}$$

$$1 + e^{-0.6114(x-69.71)} = 2$$

$$e^{-0.6114(x-69.71)} = 1$$

$$-0.6114(x - 69.71) = \ln 1$$

$$-0.6114(x - 69.71) = 0$$

$$x = 69.71 \text{ inches}$$

Females

$$50 = \frac{100}{1 + e^{-0.66607(x-64.51)}}$$

$$1 + e^{-0.66607(x-64.51)} = 2$$

$$e^{-0.6667(x-64.51)} = 1$$

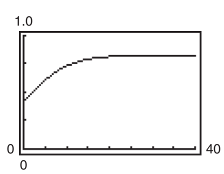
$$-0.66607(x - 64.51) = \ln 1$$

$$-0.66607(x - 64.51) = 0$$

$$x = 64.51 \text{ inches}$$

116. $P = \dfrac{0.83}{1 + e^{-0.2n}}$

(a)



(b) Horizontal asymptotes: $P = 0, P = 0.83$
The upper asymptote, $P = 0.83$, indicates that the proportion of correct responses will approach 0.83 as the number of trials increases.

(c) When $P = 60\%$ or $P = 0.60$:

$$0.60 = \frac{0.83}{1 + e^{-0.2n}}$$

$$1 + e^{-0.2n} = \frac{0.83}{0.60}$$

$$e^{-0.2n} = \frac{0.83}{0.60} - 1$$

$$\ln e^{-0.2n} = \ln\left(\frac{0.83}{0.60} - 1\right)$$

$$-0.2n = \ln\left(\frac{0.83}{0.60} - 1\right)$$

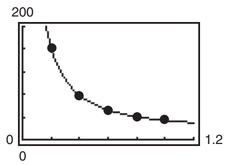
$$n = -\frac{\ln\left(\frac{0.83}{0.60} - 1\right)}{0.2} \approx 5 \text{ trials}$$

117. $y = -3.00 + 11.88 \ln x + \dfrac{36.94}{x}$

(a)

x	0.2	0.4	0.6	0.8	1.0
y	162.6	78.5	52.5	40.5	33.9

(b)



The model seems to fit the data well.

(c) When $y = 30$:

$$30 = -3.00 + 11.88 \ln x + \frac{36.94}{x}$$

Add the graph of $y = 30$ to the graph in part (a) and estimate the point of intersection of the two graphs. We find that $x \approx 1.20$ meters.

(d) No, it is probably not practical to lower the number of gs experienced during impact to less than 23 because the required distance traveled at $y = 23$ is $x \approx 2.27$ meters. It is probably not practical to design a car allowing a passenger to move forward 2.27 meters (or 7.45 feet) during an impact.

118. $T = 20[1 + 7(2^{-h})]$

(a) From the graph in the textbook we see a horizontal asymptote at $T = 20$. This represents the room temperature.

(b)
$$\begin{aligned} 100 &= 20[1 + 7(2^{-h})] \\ 5 &= 1 + 7(2^{-h}) \\ 4 &= 7(2^{-h}) \\ \frac{4}{7} &= 2^{-h} \\ \ln\left(\frac{4}{7}\right) &= \ln 2^{-h} \\ \ln\left(\frac{4}{7}\right) &= -h \ln 2 \\ \frac{\ln(4/7)}{-\ln 2} &= h \\ h &\approx 0.81 \text{ hour} \end{aligned}$$

119. $\log_a(uv) = \log_a u + \log_a v$

True by Property 1 in Section 3.3.

120. $\log_a(u + v) = (\log_a u)(\log_a v)$

False.

$2.04 \approx \log_{10}(10 + 100) \neq (\log_{10} 10)(\log_{10} 100) = 2$

121. $\log_a(u - v) = \log_a u - \log_a v$

False.

$$\begin{aligned} 1.95 &\approx \log(100 - 10) \\ &\neq \log 100 - \log 10 = 1 \end{aligned}$$

122. $\log_a\left(\dfrac{u}{v}\right) = \log_a u - \log_a v$

True by Property 2 in Section 3.3.

123. Yes, a logarithmic equation can have more than one extraneous solution. See Exercise 93.

124. $A = Pe^{rt}$

(a) $A = (2P)e^{rt} = 2(Pe^{rt})$ This doubles your money.

(b) $A = Pe^{(2r)t} = Pe^{rt}e^{rt} = e^{rt}(Pe^{rt})$

(c) $A = Pe^{r(2t)} = Pe^{rt}e^{rt} = e^{rt}(Pe^{rt})$

Doubling the interest rate yields the same result as doubling the number of years.

If $2 > e^{rt}$ (i.e., $rt < \ln 2$), then doubling your investment would yield the most money. If $rt > \ln 2$, then doubling either the interest rate or the number of years would yield more money.

125. Yes.

Time to Double	Time to Quadruple
$2P = Pe^{rt}$	$4P = Pe^{rt}$
$2 = e^{rt}$	$4 = e^{rt}$
$\ln 2 = rt$	$\ln 4 = rt$
$\dfrac{\ln 2}{r} = t$	$\dfrac{2 \ln 2}{r} = t$

Thus, the time to quadruple is twice as long as the time to double.

126. (a) When solving an exponential equation, rewrite the original equation in a form that allows you to use the One-to-One Property $a^x = a^y$ if and only if $x = y$ or rewrite the original equation in logarithmic form and use the Inverse Property $\log_a a^x = x$.

(b) When solving a logarithmic equation, rewrite the original equation in a form that allows you to use the One-to-One Property $\log_a x = \log_a y$ if and only if $x = y$ or rewrite the original equation in exponential form and use the Inverse Property $a^{\log_a x} = x$.

127. $\sqrt{48x^2y^5} = \sqrt{16x^2y^4 3y}$
$= 4|x|y^2\sqrt{3y}$

128. $\sqrt{32} - 2\sqrt{25} = \sqrt{16 \cdot 2} - 2(5)$
$= 4\sqrt{2} - 10$

129. $\sqrt[3]{25}\sqrt[3]{15} = \sqrt[3]{375}$
$= \sqrt[3]{125 \cdot 3} = 5\sqrt[3]{3}$

130.
$$\frac{3}{\sqrt{10} - 2} = \frac{3}{\sqrt{10} - 2} \cdot \frac{\sqrt{10} + 2}{\sqrt{10} + 2}$$
$$= \frac{3\left(\sqrt{10} + 2\right)}{10 - 4}$$
$$= \frac{3\left(\sqrt{10} + 2\right)}{6}$$
$$= \frac{\sqrt{10} + 2}{2}$$
$$= \frac{1}{2}\sqrt{10} + 1$$

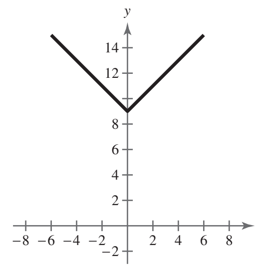
131. $f(x) = |x| + 9$

Domain: all real numbers x

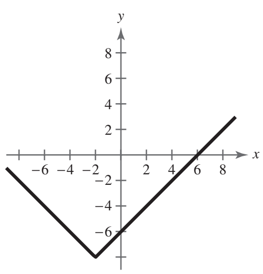
y-intercept: $(0, 9)$

y-axis symmetry

x	0	± 1	± 2	± 3
y	9	10	11	12



132.

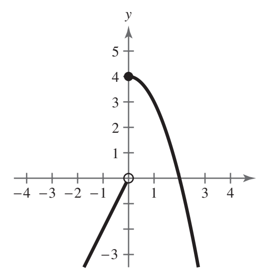


133. $g(x) = \begin{cases} 2x, & x < 0 \\ -x^2 + 4, & x \geq 0 \end{cases}$

Domain: all real numbers x

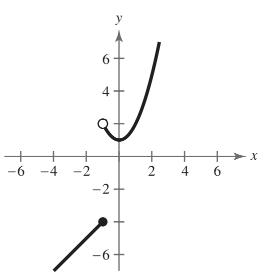
x-intercept: $(2, 0)$

y-intercept: $(0, 4)$



x	-3	-2	-1	-0.5	0	1	2	3
y	-6	-4	-2	-1	4	3	2	-5

134.



135. $\log_6 9 = \dfrac{\log_{10} 9}{\log_{10} 6} = \dfrac{\ln 9}{\ln 6} \approx 1.226$

136. $\log_3 4 = \dfrac{\log_{10} 4}{\log_{10} 3} = \dfrac{\ln 4}{\ln 3} \approx 1.262$

137. $\log_{3/4} 5 = \dfrac{\log_{10} 5}{\log_{10}(3/4)} = \dfrac{\ln 5}{\ln(3/4)} \approx -5.595$

138. $\log_8 22 = \dfrac{\log_{10} 22}{\log_{10} 8} = \dfrac{\ln 22}{\ln 8} \approx 1.486$

Section 3.5 Exponential and Logarithmic Models

- You should be able to solve growth and decay problems.
  (a) Exponential growth if $b > 0$ and $y = ae^{bx}$.
  (b) Exponential decay if $b > 0$ and $y = ae^{-bx}$.
- You should be able to use the Gaussian model
  $y = ae^{-(x-b)^2/c}$.
- You should be able to use the logistic growth model
  $y = \dfrac{a}{1 + be^{-rx}}$.
- You should be able to use the logarithmic models
  $y = a + b \ln x$, $y = a + b \log x$.

Vocabulary Check

1. $y = ae^{bx}$; $y = ae^{-bx}$ **2.** $y = a + b \ln x$; $y = a + b \log x$ **3.** normally distributed

4. bell; average value **5.** sigmoidal

1. $y = 2e^{x/4}$

This is an exponential growth model. Matches graph (c).

2. $y = 6e^{-x/4}$

This is an exponential decay model. Matches graph (e).

3. $y = 6 + \log(x + 2)$

This is a logarithmic function shifted up six units and left two units. Matches graph (b).

4. $y = 3e^{-(x-2)^2/5}$

This is a Gaussian model. Matches graph (a).

5. $y = \ln(x + 1)$

This is a logarithmic model shifted left one unit. Matches graph (d).

6. $y = \dfrac{4}{1 + e^{-2x}}$

This is a logistic growth model. Matches graph (f).

7. Since $A = 1000e^{0.035t}$, the time to double is given by $2000 = 1000e^{0.035t}$ and we have

$$2 = e^{0.035t}$$
$$\ln 2 = \ln e^{0.035t}$$
$$\ln 2 = 0.035t$$
$$t = \frac{\ln 2}{0.035} \approx 19.8 \text{ years.}$$

Amount after 10 years: $A = 1000e^{0.35} \approx \$1419.07$

8. Since $A = 750e^{0.105t}$, the time to double is given by $1500 = 750e^{0.105t}$, and we have

$$1500 = 750e^{0.105t}$$
$$2 = e^{0.105t}$$
$$\ln 2 = \ln e^{0.105t}$$
$$\ln 2 = 0.105t$$
$$t = \frac{\ln 2}{0.105} \approx 6.60 \text{ years.}$$

Amount after 10 years: $A = 750e^{0.105(10)} \approx \$2143.24$

9. Since $A = 750e^{rt}$ and $A = 1500$ when $t = 7.75$, we have the following.

$$1500 = 750e^{7.75r}$$
$$2 = e^{7.75r}$$
$$\ln 2 = \ln e^{7.75r}$$
$$\ln 2 = 7.75r$$
$$r = \frac{\ln 2}{7.75} \approx 0.089438 = 8.9438\%$$

Amount after 10 years: $A = 750e^{0.089438(10)} \approx \$1834.37$

10. Since $A = 10{,}000e^{rt}$ and $A = 20{,}000$ when $t = 12$, we have

$$20{,}000 = 10{,}000e^{12r}$$
$$2 = e^{12r}$$
$$\ln 2 = \ln e^{12r}$$
$$\ln 2 = 12r$$
$$r = \frac{\ln 2}{12} \approx 0.057762 = 5.7762\%.$$

Amount after 10 years:

$A = 10{,}000e^{0.057762(10)} \approx \$17{,}817.97$

11. Since $A = 500e^{rt}$ and $A = \$1505.00$ when $t = 10$, we have the following.

$$1505.00 = 500e^{10r}$$
$$r = \frac{\ln(1505.00/500)}{10} \approx 0.110 = 11.0\%$$

The time to double is given by

$$1000 = 500e^{0.110t}$$
$$t = \frac{\ln 2}{0.110} \approx 6.3 \text{ years.}$$

12. Since $A = 600e^{rt}$ and $A = 19{,}205$ when $t = 10$, we have

$$19{,}205 = 600e^{10r}$$
$$\frac{19{,}205}{600} = e^{10r}$$
$$\ln\left(\frac{19{,}205}{600}\right) = \ln e^{10r}$$
$$\ln\left(\frac{19{,}205}{600}\right) = 10r$$
$$r = \frac{\ln(19{,}205/600)}{10} \approx 0.3466 \text{ or } 34.66\%.$$

The time to double is given by

$$1200 = 600e^{0.3466t}$$
$$t = \frac{\ln 2}{0.3466} \approx 2 \text{ years.}$$

13. Since $A = Pe^{0.045t}$ and $A = 10{,}000.00$ when $t = 10$, we have the following.

$$10{,}000.00 = Pe^{0.045(10)}$$
$$\frac{10{,}000.00}{e^{0.045(10)}} = P \approx \$6376.28$$

The time to double is given by $t = \dfrac{\ln 2}{0.045} \approx 15.40$ years.

14. Since $A = Pe^{0.02t}$ and $A = 2000$ when $t = 10$, we have

$$2000 = Pe^{0.02(10)}$$
$$P = \frac{2000}{e^{0.02(10)}} = \$1637.46.$$

The time to double is given by $t = \dfrac{\ln 2}{0.02} = 34.7$ years.

15. $500{,}000 = P\left(1 + \dfrac{0.075}{12}\right)^{12(20)}$

$$P = \frac{500{,}000}{\left(1 + \frac{0.075}{12}\right)^{12(20)}}$$
$$= \frac{500{,}000}{1.00625^{240}} \approx \$112{,}087.09$$

16.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$500{,}000 = P\left(1 + \frac{0.12}{12}\right)^{12(40)}$$
$$P = \$4214.16$$

17. $P = 1000,\ r = 11\%$

(a) $n = 1$

$$(1 + 0.11)^t = 2$$
$$t \ln 1.11 = \ln 2$$
$$t = \frac{\ln 2}{\ln 1.11} \approx 6.642 \text{ years}$$

(b) $n = 12$

$$\left(1 + \frac{0.11}{12}\right)^{12t} = 2$$
$$12t \ln\left(1 + \frac{0.11}{12}\right) = \ln 2$$
$$t = \frac{\ln 2}{12 \ln\left(1 + \frac{0.11}{12}\right)} \approx 6.330 \text{ years}$$

(c) $n = 365$

$$\left(1 + \frac{0.11}{365}\right)^{365t} = 2$$
$$365t \ln\left(1 + \frac{0.11}{365}\right) = \ln 2$$
$$t = \frac{\ln 2}{365 \ln\left(1 + \frac{0.11}{365}\right)} \approx 6.302 \text{ years}$$

(d) Compounded continuously

$$e^{0.11t} = 2$$
$$0.11t = \ln 2$$
$$t = \frac{\ln 2}{0.11} \approx 6.301 \text{ years}$$

18. $P = 1000,\ r = 10.5\% = 0.105$

(a) $n = 1$

$$t = \frac{\ln 2}{\ln(1 + 0.105)} \approx 6.94 \text{ years}$$

(b) $n = 12$

$$t = \frac{\ln 2}{12 \ln\left(1 + \frac{0.105}{12}\right)} \approx 6.63 \text{ years}$$

(c) $n = 365$

$$t = \frac{\ln 2}{365 \ln\left(1 + \frac{0.105}{365}\right)} \approx 6.602 \text{ years}$$

(d) Compounded continuously

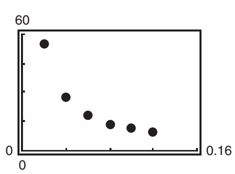
$$t = \frac{\ln 2}{0.105} \approx 6.601 \text{ years}$$

19.

$$3P = Pe^{rt}$$
$$3 = e^{rt}$$
$$\ln 3 = rt$$
$$\frac{\ln 3}{r} = t$$

r	2%	4%	6%	8%	10%	12%
$t = \frac{\ln 3}{r}$ (years)	54.93	27.47	18.31	13.73	10.99	9.16

20.

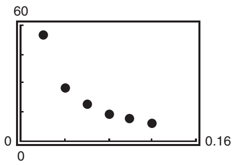


Using the power regression feature of a graphing utility, $t = 1.099r^{-1}$.

21.

$$3P = P(1 + r)^t$$
$$3 = (1 + r)^t$$
$$\ln 3 = \ln(1 + r)^t$$
$$\ln 3 = t \ln(1 + r)$$
$$\frac{\ln 3}{\ln(1 + r)} = t$$

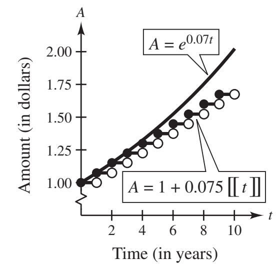
r	2%	4%	6%	8%	10%	12%
$t = \frac{\ln 3}{\ln(1 + r)}$ (years)	55.48	28.01	18.85	14.27	11.53	9.69

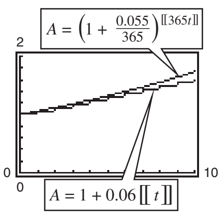
22. 

Using the power regression feature of a graphing utility, $t = 1.222r^{-1}$.

23. Continuous compounding results in faster growth.

$A = 1 + 0.075[\![t]\!]$ and $A = e^{0.07t}$



24. 

From the graph, $5\frac{1}{2}\%$ compounded daily grows faster than 6% simple interest.

25. $\frac{1}{2}C = Ce^{k(1599)}$

$0.5 = e^{k(1599)}$

$\ln 0.5 = \ln e^{k(1599)}$

$\ln 0.5 = k(1599)$

$k = \dfrac{\ln 0.5}{1599}$

Given $C = 10$ grams after 1000 years, we have

$y = 10e^{[(\ln 0.5)/1599](1000)}$

≈ 6.48 grams.

26. $\frac{1}{2}C = Ce^{k(1599)}$

$\frac{1}{2} = e^{k(1599)}$

$\ln \frac{1}{2} = \ln e^{k(1599)}$

$\ln \frac{1}{2} = k(1599)$

$k = \dfrac{\ln(1/2)}{1599}$

Given $y = 1.5$ grams after 1000 years, we have

$1.5 = Ce^{[\ln(1/2)/1599](1000)}$

$C \approx 2.31$ grams.

27. $\frac{1}{2}C = Ce^{k(5715)}$

$0.5 = e^{k(5715)}$

$\ln 0.5 = \ln e^{k(5715)}$

$\ln 0.5 = k(5715)$

$k = \dfrac{\ln 0.5}{5715}$

Given $y = 2$ grams after 1000 years, we have

$2 = Ce^{[(\ln 0.5)/5715](1000)}$

$C \approx 2.26$ grams.

28. $\frac{1}{2}C = Ce^{k(5715)}$

$\frac{1}{2} = e^{k(5715)}$

$\ln \frac{1}{2} = \ln e^{k(5715)}$

$\ln \frac{1}{2} = k(5715)$

$k = \dfrac{\ln(1/2)}{5715}$

Given $C = 3$ grams, after 1000 years we have

$y = 3e^{[\ln(1/2)/5715](1000)}$

$y \approx 2.66$ grams.

29. $\frac{1}{2}C = Ce^{k(24{,}100)}$

$0.5 = e^{k(24{,}100)}$

$\ln 0.5 = \ln e^{k(24{,}100)}$

$\ln 0.5 = k(24{,}100)$

$k = \dfrac{\ln 0.5}{24{,}100}$

Given $y = 2.1$ grams after 1000 years, we have

$2.1 = Ce^{[(\ln 0.5)/24{,}100](1000)}$

$C \approx 2.16$ grams.

30. $\frac{1}{2}C = Ce^{k(24,100)}$

$\frac{1}{2} = e^{k(24,100)}$

$\ln\frac{1}{2} = \ln e^{k(24,100)}$

$\ln\frac{1}{2} = k(24,100)$

$k = \frac{\ln(1/2)}{24,100}$

Given $y = 0.4$ grams after 1000 years, we have

$0.4 = Ce^{[\ln(1/2)/24,100](1000)}$

$C \approx 0.41$ grams.

31. $y = ae^{bx}$

$1 = ae^{b(0)} \implies 1 = a$

$10 = e^{b(3)}$

$\ln 10 = 3b$

$\frac{\ln 10}{3} = b \implies b \approx 0.7675$

Thus, $y = e^{0.7675x}$.

32. $y = ae^{bx}$

$\frac{1}{2} = ae^{b(0)} \implies a = \frac{1}{2}$

$5 = \frac{1}{2}e^{b(4)}$

$10 = e^{4b}$

$\ln 10 = \ln e^{4b}$

$\ln 10 = 4b$

$\frac{\ln 10}{4} = b \implies b \approx 0.5756$

Thus, $y = \frac{1}{2}e^{0.5756x}$.

33. $y = ae^{bx}$

$5 = ae^{b(0)} \implies 5 = a$

$1 = 5e^{b(4)}$

$\frac{1}{5} = e^{4b}$

$\ln\left(\frac{1}{5}\right) = 4b$

$\frac{\ln(1/5)}{4} = b \implies b \approx -0.4024$

Thus, $y = 5e^{-0.4024x}$.

34. $y = ae^{bx}$

$1 = ae^{b(0)} \implies 1 = a$

$\frac{1}{4} = e^{b(3)}$

$\ln\left(\frac{1}{4}\right) = \ln e^{3b}$

$\ln\left(\frac{1}{4}\right) = 3b$

$\frac{\ln(1/4)}{3} = b \implies b \approx -0.4621$

Thus, $y = e^{-0.4621x}$.

35. $P = 2430e^{-0.0029t}$

(a) Since the exponent is negative, this is an exponential decay model. The population is decreasing.

(b) For 2000, let $t = 0$: $P = 2430$ thousand people

For 2003, let $t = 3$: $P \approx 2408.95$ thousand people

(c) 2.3 million $=$ 2300 thousand

$$2300 = 2430e^{-0.0029t}$$

$$\frac{2300}{2430} = e^{-0.0029t}$$

$$\ln\left(\frac{2300}{2430}\right) = -0.0029t$$

$$t = \frac{\ln(2300/2430)}{-0.0029} \approx 18.96$$

The population will reach 2.3 million (according to the model) during the later part of the year 2018.

36.

Country	2000	2010
Bulgaria	7.8	7.1
Canada	31.3	34.3
China	1268.9	1347.6
United Kingdom	59.5	61.2
United States	282.3	309.2

—CONTINUED—

36. —CONTINUED—

(a) Bulgaria:

$$a = 7.8$$
$$7.1 = 7.8e^{b(10)}$$
$$\ln\frac{7.1}{7.8} = 10b \implies b = -0.0094$$

For 2030, use $t = 30$.

$$y = 7.8e^{-0.0094(30)} = 5.88 \text{ million}$$

China:

$$a = 1268.9$$
$$1347.6 = 1268.9e^{b(10)}$$
$$\ln\frac{1347.6}{1268.9} = 10b \implies b \approx 0.00602$$

For 2030, use $t = 30$.

$$y = 1268.9e^{0.00602(30)} \approx 1520.06 \text{ million}$$

United Kingdom:

$$a = 59.5$$
$$61.2 = 59.5e^{b(10)}$$
$$\ln\frac{61.2}{59.5} = 10b \implies b \approx 0.00282$$

For 2030, use $t = 30$.

$$y = 59.5e^{0.00282(30)} \approx 64.7 \text{ million}$$

Canada:

$$a = 31.3$$
$$34.3 = 31.3e^{b(10)}$$
$$\ln\frac{34.3}{31.3} = 10b \implies b \approx 0.00915$$

For 2030, use $t = 30$.

$$y = 31.3e^{0.00915(30)} \approx 41.2 \text{ million}$$

United States:

$$a = 282.3$$
$$309.2 = 282.3e^{b(10)}$$
$$\ln\frac{309.2}{282.3} = 10b \implies b \approx 0.0091$$

For 2030, use $t = 30$.

$$y = 282.3e^{0.0091(30)} \approx 370.9 \text{ million}$$

(b) The constant b determines the growth rates. The greater the rate of growth, the greater the value of b.

(c) The constant b determines whether the population is increasing $(b > 0)$ or decreasing $(b < 0)$.

37. $y = 4080e^{kt}$

When $t = 3$, $y = 10{,}000$:

$$10{,}000 = 4080e^{k(3)}$$
$$\frac{10{,}000}{4080} = e^{3k}$$
$$\ln\left(\frac{10{,}000}{4080}\right) = 3k$$
$$k = \frac{\ln(10{,}000/4080)}{3} \approx 0.2988$$

When $t = 24$: $y = 4080e^{0.2988(24)} \approx 5{,}309{,}734$ hits

38.

$$y = 10e^{kt}$$
$$65 = 10e^{k(14)}$$
$$\ln\left(\frac{65}{10}\right) = 14k \implies k = 0.1337$$

For 2010, $t = 20$:

$$y = 10e^{(0.1337)(20)} = \$144.98 \text{ million}$$

39. $N = 100e^{kt}$

$300 = 100e^{5k}$

$3 = e^{5k}$

$\ln 3 = \ln e^{5k}$

$\ln 3 = 5k$

$k = \dfrac{\ln 3}{5} \approx 0.2197$

$N = 100e^{0.2197t}$

$200 = 100e^{0.2197t}$

$t = \dfrac{\ln 2}{0.2197} \approx 3.15$ hours

40. $N = 250e^{kt}$

$280 = 250e^{k(10)}$

$1.12 = e^{10k}$

$k = \dfrac{\ln 1.12}{10}$

$N = 250e^{[(\ln 1.12)/10]t}$

$500 = 250e^{[(\ln 1.12)/10]t}$

$2 = e^{[(\ln 1.12)/10]t}$

$\ln 2 = \left(\dfrac{\ln 1.12}{10}\right)t$

$t = \dfrac{\ln 2}{(\ln 1.12)/10} \approx 61.16$ hours

41. $R = \dfrac{1}{10^{12}}e^{-t/8223}$

(a) $R = \dfrac{1}{8^{14}}$

$\dfrac{1}{10^{12}}e^{-t/8223} = \dfrac{1}{8^{14}}$

$e^{-t/8223} = \dfrac{10^{12}}{8^{14}}$

$-\dfrac{t}{8223} = \ln\left(\dfrac{10^{12}}{8^{14}}\right)$

$t = -8223 \ln\left(\dfrac{10^{12}}{8^{14}}\right) \approx 12{,}180$ years old

(b) $\dfrac{1}{10^{12}}e^{-t/8223} = \dfrac{1}{13^{11}}$

$e^{-t/8223} = \dfrac{10^{12}}{13^{11}}$

$-\dfrac{t}{8223} = \ln\left(\dfrac{10^{12}}{13^{11}}\right)$

$t = -8223 \ln\left(\dfrac{10^{12}}{13^{11}}\right) \approx 4797$ years old

42. $y = Ce^{kt}$

$\dfrac{1}{2}C = Ce^{5715k}$

$\ln \dfrac{1}{2} = 5715k$

$k = \dfrac{\ln(1/2)}{5715}$

The ancient charcoal has only 15% as much radioactive carbon.

$0.15C = Ce^{[(\ln 0.5)/5715]t}$

$\ln 0.15 = \dfrac{\ln 0.5}{5715}t$

$t = \dfrac{5715 \ln 0.15}{\ln 0.5} \approx 15{,}642$ years

43. (0, 30,788), (2, 18,000)

(a) $m = \dfrac{18{,}000 - 30{,}788}{2 - 0} = -6394$

$b = 30{,}788$

Linear model:
$V = -6394t + 30{,}788$

(b) $a = 30{,}788$

$18{,}000 = 30{,}788e^{k(2)}$

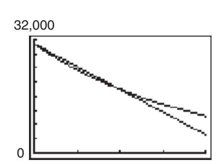
$\dfrac{4500}{7697} = e^{2k}$

$\ln\left(\dfrac{4500}{7697}\right) = 2k$

$k = \dfrac{1}{2}\ln\left(\dfrac{4500}{7697}\right) \approx -0.268$

Exponential model: $V = 30{,}788e^{-0.268t}$

(c)



The exponential model depreciates faster in the first two years.

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43. —CONTINUED—

(d)

t	1	3
$V = -6394t + 30{,}788$	\$24,394	\$11,606
$V = 30{,}788e^{-0.268t}$	\$23,550	\$13,779

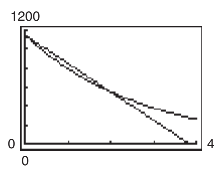
(e) The linear model gives a higher value for the car for the first two years, then the exponential model yields a higher value. If the car is less than two years old, the seller would most likely want to use the linear model and the buyer the exponential model. If it is more than two years old, the opposite is true.

44. (0, 1150), (2, 550)

(a) $m = \dfrac{550 - 1150}{2 - 0} = -300$

$V = -300t + 1150$

(c)



The exponential model depreciates faster in the first two years.

(e) The slope of the linear model means that the computer depreciates \$300 per year, then loses all value in the third year. The exponential model depreciates faster in the first two years but maintains value longer.

(b) $550 = 1150e^{k(2)}$

$\ln\left(\dfrac{550}{1150}\right) = 2k \implies k \approx -0.369$

$V = 1150e^{-0.369t}$

(d)

t	1	3
$V = -300t + 1100$	\$850	\$250
$V = 1150e^{-0.369t}$	\$795	\$380

45. $S(t) = 100(1 - e^{kt})$

(a) $15 = 100(1 - e^{k(1)})$

$-85 = -100e^k$

$\frac{85}{100} = e^k$

$0.85 = e^k$

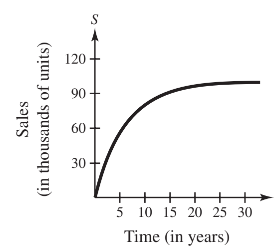
$\ln 0.85 = \ln e^k$

$k = \ln 0.85$

$k \approx -0.1625$

$S(t) = 100(1 - e^{-0.1625t})$

(b)



(c) $S(5) = 100(1 - e^{-0.1625(5)}) \approx 55.625 = 55{,}625$ units

46. $N = 30(1 - e^{kt})$

(a) $N = 19, t = 20$

$19 = 30(1 - e^{20k})$

$30e^{20k} = 11$

$e^{20k} = \dfrac{11}{30}$

$\ln e^{20k} = \ln\left(\dfrac{11}{30}\right)$

$20k = \ln\left(\dfrac{11}{30}\right)$

$k = -0.050$

So, $N = 30(1 - e^{-0.050})$.

(b) $N = 25$

$25 = 30(1 - e^{-0.050t})$

$\dfrac{5}{30} = e^{-0.050t}$

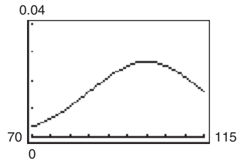
$\ln\left(\dfrac{5}{30}\right) = \ln e^{-0.050t}$

$\ln\left(\dfrac{5}{30}\right) = -0.050t$

$t = \dfrac{\ln(5/30)}{-0.050} = 36$ days

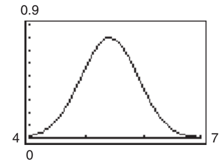
47. $y = 0.0266e^{-(x-100)^2/450}$, $70 \le x \le 116$

(a)



(b) The average IQ score of an adult student is 100.

48. (a)



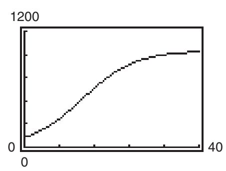
(b) The average number of hours per week a student uses the tutor center is 5.4.

49. $p(t) = \dfrac{1000}{1 + 9e^{-0.1656t}}$

(a) $p(5) = \dfrac{1000}{1 + 9e^{-0.1656(5)}} \approx 203$ animals

(b)
$$500 = \frac{1000}{1 + 9e^{-0.1656t}}$$
$$1 + 9e^{-0.1656t} = 2$$
$$9e^{-0.1656t} = 1$$
$$e^{-0.1656t} = \frac{1}{9}$$
$$t = -\frac{\ln(1/9)}{0.1656} \approx 13 \text{ months}$$

(c)



The horizontal asymptotes are $p = 0$ and $p = 1000$. The asymptote with the larger p-value, $p = 1000$, indicates that the population size will approach 1000 as time increases.

50. $S = \dfrac{500{,}000}{1 + 0.6e^{kt}}$

(a)
$$300{,}000 = \frac{500{,}000}{1 + 0.6e^{4k}}$$
$$1 + 0.6e^{4k} = \frac{5}{3}$$
$$0.6e^{4k} = \frac{2}{3}$$
$$e^{4k} = \frac{10}{9}$$
$$4k = \ln\left(\frac{10}{9}\right)$$
$$k = \frac{1}{4}\ln\left(\frac{10}{9}\right) \approx 0.0263$$

So, $S = \dfrac{500{,}000}{1 + 0.6e^{0.0263t}}$.

(b) When $t = 8$:
$$S = \frac{500{,}000}{1 + 0.6e^{0.0263(8)}} \approx 287{,}273 \text{ units sold.}$$

51. $R = \log \dfrac{I}{I_0} = \log I$ since $I_0 = 1$.

(a) $7.9 = \log I \implies I = 10^{7.9} \approx 79{,}432{,}823$

(b) $8.3 = \log I \implies I = 10^{8.3} \approx 199{,}526{,}231$

(c) $4.2 = \log I \implies I = 10^{4.2} \approx 15{,}849$

52. $R = \log \dfrac{I}{I_0} = \log I$ since $I_0 = 1$.

(a) $R = \log 80{,}500{,}000 = 7.91$

(b) $R = \log 48{,}275{,}000 = 7.68$

(c) $R = \log 251{,}200 = 5.40$

53. $\beta = 10 \log \dfrac{I}{I_0}$ where $I_0 = 10^{-12}$ watt/m^2.

(a) $\beta = 10 \log \dfrac{10^{-10}}{10^{-12}} = 10 \log 10^2 = 20$ decibels

(b) $\beta = 10 \log \dfrac{10^{-5}}{10^{-12}} = 10 \log 10^7 = 70$ decibels

(c) $\beta = 10 \log \dfrac{10^{-8}}{10^{-12}} = 10 \log 10^4 = 40$ decibels

(d) $\beta = 10 \log \dfrac{1}{10^{-12}} = 10 \log 10^{12} = 120$ decibels

54. $\beta(I) = 10 \log \dfrac{I}{I_0}$ where $I_0 = 10^{-12}$ watt/m^2

(a) $\beta(10^{-11}) = 10 \log \dfrac{10^{-11}}{10^{-12}} = 10 \log 10^1 = 10$ decibels

(b) $\beta(10^{2}) = 10 \log \dfrac{10^{2}}{10^{-12}} = 10 \log 10^{14} = 140$ decibels

(c) $\beta(10^{-4}) = 10 \log \dfrac{10^{-4}}{10^{-12}} = 10 \log 10^8 = 80$ decibels

(d) $\beta(10^{-2}) = 10 \log \dfrac{10^{-2}}{10^{-12}} = 10 \log 10^{10} = 100$ decibels

55.
$$\begin{aligned}
\beta &= 10 \log \frac{I}{I_0} \\
\frac{\beta}{10} &= \log \frac{I}{I_0} \\
10^{\beta/10} &= 10^{\log I/I_0} \\
10^{\beta/10} &= \frac{I}{I_0} \\
I &= I_0 10^{\beta/10} \\
\%\text{ decrease} &= \frac{I_0 10^{9.3} - I_0 10^{8.0}}{I_0 10^{9.3}} \times 100 \approx 95\%
\end{aligned}$$

56.
$$\begin{aligned}
\beta &= 10 \log_{10} \frac{I}{I_0} \\
10^{\beta/10} &= \frac{I}{I_0} \\
I &= I_0 10^{\beta/10} \\
\%\text{ decrease} &= \frac{I_0 10^{8.8} - I_0 10^{7.2}}{I_0 10^{8.8}} \times 100 \approx 97\%
\end{aligned}$$

57. $\text{pH} = -\log[\text{H}^+]$

$-\log(2.3 \times 10^{-5}) \approx 4.64$

58. $\text{pH} = -\log[\text{H}^+]$

$-\log[11.3 \times 10^{-6}] \approx 4.95$

59.
$$\begin{aligned}
5.8 &= -\log[\text{H}^+] \\
-5.8 &= \log[\text{H}^+] \\
10^{-5.8} &= 10^{\log[\text{H}^+]} \\
10^{-5.8} &= [\text{H}^+]
\end{aligned}$$

$[\text{H}^+] \approx 1.58 \times 10^{-6}$ mole per liter

60.
$$\begin{aligned}
3.2 &= -\log[\text{H}^+] \\
10^{-3.2} &= [\text{H}^+]
\end{aligned}$$

$[\text{H}^+] \approx 6.3 \times 10^{-4}$ mole per liter

61.
$$\begin{aligned}
2.9 &= -\log[\text{H}^+] \\
-2.9 &= \log[\text{H}^+]
\end{aligned}$$

$[\text{H}^+] = 10^{-2.9}$ for the apple juice

$$\begin{aligned}
8.0 &= -\log[\text{H}^+] \\
-8.0 &= \log[\text{H}^+]
\end{aligned}$$

$[\text{H}^+] = 10^{-8}$ for the drinking water

$\dfrac{10^{-2.9}}{10^{-8}} = 10^{5.1}$ times the hydrogen ion concentration of drinking water

62.
$$\begin{aligned}
\text{pH} - 1 &= -\log[\text{H}^+] \\
-(\text{pH} - 1) &= \log[\text{H}^+] \\
10^{-(\text{pH}-1)} &= [\text{H}^+] \\
10^{-\text{pH}+1} &= [\text{H}^+] \\
10^{-\text{pH}} \cdot 10 &= [\text{H}^+]
\end{aligned}$$

The hydrogen ion concentration is increased by a factor of 10.

63. $t = -10 \ln \dfrac{T - 70}{98.6 - 70}$

At 9:00 A.M. we have:

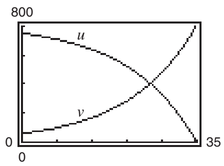
$t = -10 \ln \dfrac{85.7 - 70}{98.6 - 70} \approx 6$ hours

From this you can conclude that the person died at 3:00 A.M.

64. Interest: $u = M - \left(M - \frac{Pr}{12}\right)\left(1 + \frac{r}{12}\right)^{12t}$

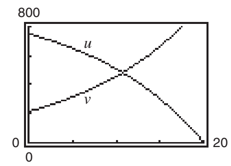
Principal: $v = \left(M - \frac{Pr}{12}\right)\left(1 + \frac{r}{12}\right)^{12t}$

(a) $P = 120{,}000,\ t = 35,\ r = 0.075,\ M = 809.39$



(b) In the early years of the mortgage, the majority of the monthly payment goes toward interest. The principal and interest are nearly equal when $t \approx 26$ years.

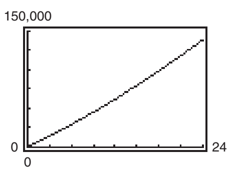
(c) $P = 120{,}000,\ t = 20,\ r = 0.075,\ M = 966.71$



The interest is still the majority of the monthly payment in the early years. Now the principal and interest are nearly equal when $t \approx 10.729 \approx 11$ years.

65. $u = 120{,}000\left[\frac{0.075t}{1 - \left(\frac{1}{1 + 0.075/12}\right)^{12t}} - 1\right]$

(a)



(b) From the graph, $u = \$120{,}000$ when $t \approx 21$ years. It would take approximately 37.6 years to pay $240,000 in interest. Yes, it is possible to pay twice as much in interest charges as the size of the mortgage. It is especially likely when the interest rates are higher.

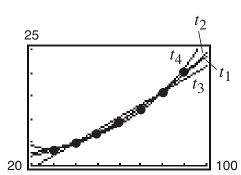
66. $t_1 = 40.757 + 0.556s - 15.817 \ln s$

$t_2 = 1.2259 + 0.0023s^2$

(a) Linear model: $t_3 = 0.2729s - 6.0143$

Exponential model: $t_4 = 1.5385e^{0.02913s}$ or $t_4 = 1.5385(1.0296)^s$

(b)



(c)

s	30	40	50	60	70	80	90
t_1	3.6	4.6	6.7	9.4	12.5	15.9	19.6
t_2	3.3	4.9	7.0	9.5	12.5	15.9	19.9
t_3	2.2	4.9	7.6	10.4	13.1	15.8	18.5
t_4	3.7	4.9	6.6	8.8	11.8	15.8	21.2

Note: Table values will vary slightly depending on the model used for t_4.

(d) Model t_1: $S_1 = |3.4 - 3.6| + |5 - 4.6| + |7 - 6.7| + |9.3 - 9.4| + |12 - 12.5| + |15.8 - 15.9| + |20 - 19.6| = 2.0$

Model t_2: $S_2 = |3.4 - 3.3| + |5 - 4.9| + |7 - 7| + |9.3 - 9.5| + |12 - 12.5| + |15.8 - 15.9| + |20 - 19.9| = 1.1$

Model t_3: $S_3 = |3.4 - 2.2| + |5 - 4.9| + |7 - 7.6| + |9.3 - 10.4| + |12 - 13.1| + |15.8 - 15.8| + |20 - 18.5| = 5.6$

Model t_4: $S_4 = |3.4 - 3.7| + |5 - 4.9| + |7 - 6.6| + |9.3 - 8.9| + |12 - 11.9| + |15.8 - 15.9| + |20 - 21.2| = 2.6$

The quadratic model, t_2, best fits the data.

67. False. The domain can be the set of real numbers for a logistic growth function.

68. False. A logistic growth function never has an x-intercept.

69. False. The graph of $f(x)$ is the graph of $g(x)$ shifted upward five units.

70. True. Powers of e are always positive, so if $a > 0$, a Gaussian model will always be greater than 0, and if $a < 0$, a Gaussian model will always be less than 0.

71. (a) Logarithmic

(b) Logistic

(c) Exponential (decay)

(d) Linear

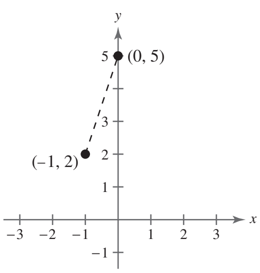
(e) None of the above (appears to be a combination of a linear and a quadratic)

(f) Exponential (growth)

72. Answers will vary.

73. $(-1, 2), (0, 5)$

(a)



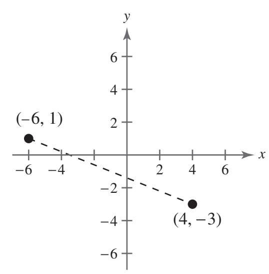
(b) $d = \sqrt{(0 - (-1))^2 + (5 - 2)^2} = \sqrt{1^2 + 3^2} = \sqrt{10}$

(c) Midpoint: $\left(\dfrac{-1 + 0}{2}, \dfrac{2 + 5}{2}\right) = \left(-\dfrac{1}{2}, \dfrac{7}{2}\right)$

(d) $m = \dfrac{5 - 2}{0 - (-1)} = \dfrac{3}{1} = 3$

74. $(4, -3), (-6, 1)$

(a)



(b) $d = \sqrt{(-6 - 4)^2 + (1 - (-3))^2}$

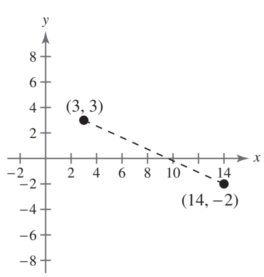
$= \sqrt{100 + 16} = \sqrt{116} = 2\sqrt{29}$

(c) Midpoint: $\left(\dfrac{-6 + 4}{2}, \dfrac{-3 + 1}{2}\right) = (-1, -1)$

(d) $m = \dfrac{-3 - 1}{4 - (-6)} = \dfrac{-4}{10} = -\dfrac{2}{5}$

75. $(3, 3), (14, -2)$

(a)



(b) $d = \sqrt{(14 - 3)^2 + (-2 - 3)^2}$

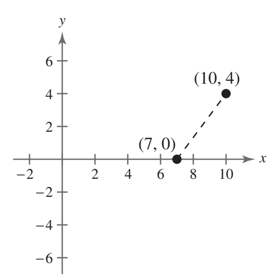
$= \sqrt{11^2 + (-5)^2} = \sqrt{146}$

(c) Midpoint: $\left(\dfrac{3 + 14}{2}, \dfrac{3 + (-2)}{2}\right) = \left(\dfrac{17}{2}, \dfrac{1}{2}\right)$

(d) $m = \dfrac{-2 - 3}{14 - 3} = -\dfrac{5}{11}$

76. $(10, 4), (7, 0)$

(a)



(b) $d = \sqrt{(10 - 7)^2 + (4 - 0)^2}$

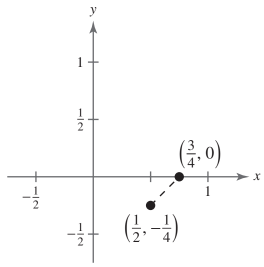
$= \sqrt{9 + 16} = \sqrt{25} = 5$

(c) Midpoint: $\left(\dfrac{7 + 10}{2}, \dfrac{0 + 4}{2}\right) = \left(\dfrac{17}{2}, 2\right)$

(d) $m = \dfrac{4 - 0}{10 - 7} = \dfrac{4}{3}$

77. $\left(\frac{1}{2}, -\frac{1}{4}\right), \left(\frac{3}{4}, 0\right)$

(a)



(b) $d = \sqrt{\left(\frac{3}{4} - \frac{1}{2}\right)^2 + \left(0 - \left(-\frac{1}{4}\right)\right)^2}$

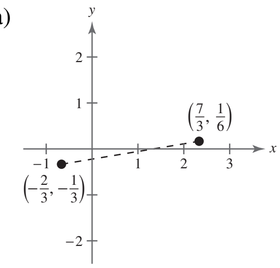
$= \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2} = \sqrt{\frac{1}{8}}$

(c) Midpoint: $\left(\frac{(1/2) + (3/4)}{2}, \frac{(-1/4) + 0}{2}\right) = \left(\frac{5}{8}, -\frac{1}{8}\right)$

(d) $m = \frac{0 - (-1/4)}{(3/4) - (1/2)} = \frac{1/4}{1/4} = 1$

78. $\left(\frac{7}{3}, \frac{1}{6}\right), \left(-\frac{2}{3}, -\frac{1}{3}\right)$

(a)



(b) $d = \sqrt{\left(-\frac{2}{3} - \frac{7}{3}\right)^2 + \left(-\frac{1}{3} - \frac{1}{6}\right)^2}$

$= \sqrt{(-3)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{9.25}$

(c) Midpoint:

$\left(\frac{(-2/3) + (7/3)}{2}, \frac{(-1/3) + (1/6)}{2}\right) = \left(\frac{5}{6}, -\frac{1}{12}\right)$

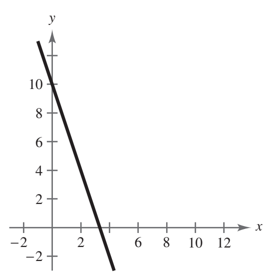
(d) $m = \frac{(-1/3) - (1/6)}{(-2/3) - (7/3)} = \frac{-1/2}{-3} = \frac{1}{6}$

79. $y = 10 - 3x$

Line

Slope: $m = -3$

y-intercept: $(0, 10)$

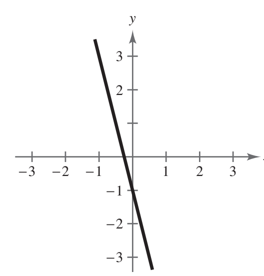


80. $y = -4x - 1$

Line

Slope: $m = -4$

y-intercept: $(0, -1)$

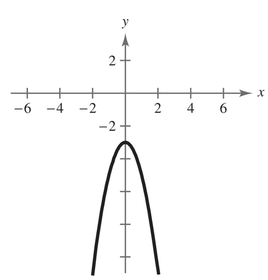


81. $y = -2x^2 - 3$

$y = -2(x - 0)^2 - 3$

Parabola

Vertex: $(0, -3)$



82. $y = 2x^2 - 7x - 30$

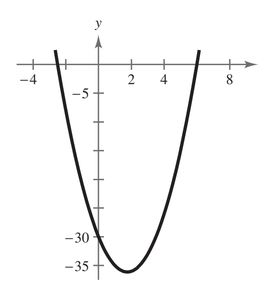
$= (2x + 5)(x - 6)$

$= 2\left(x - \frac{7}{4}\right)^2 - \frac{289}{8}$

Parabola

Vertex: $\left(\frac{7}{4}, -\frac{289}{8}\right)$

x-intercepts: $\left(-\frac{5}{2}, 0\right), (6, 0)$



83. $3x^2 - 4y = 0$

$3x^2 = 4y$

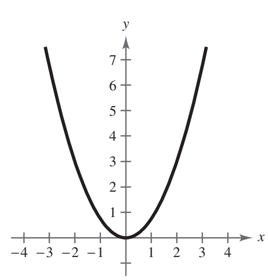
$x^2 = \frac{4}{3}y$

Parabola

Vertex: $(0, 0)$

Focus: $\left(0, \frac{1}{3}\right)$

Directrix: $y = -\frac{1}{3}$



84. $-x^2 - 8y = 0$

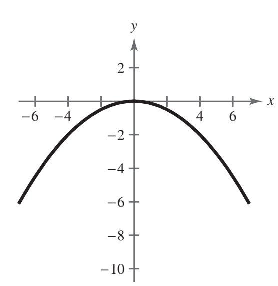
$x^2 = -8y$

Parabola

Vertex: $(0, 0)$

Focus: $(0, -2)$

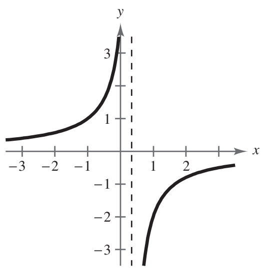
Directrix: $y = 2$



85. $y = \dfrac{4}{1 - 3x}$

Vertical asymptote: $x = \dfrac{1}{3}$

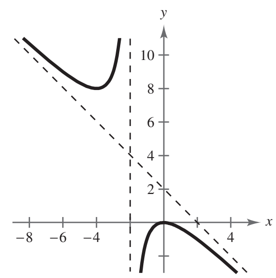
Horizontal asymptote: $y = 0$



86. $y = \dfrac{x^2}{-x - 2} = -x + 2 + \dfrac{4}{-x - 2}$

Vertical asymptote: $x = -2$

Slant asymptote: $y = -x + 2$

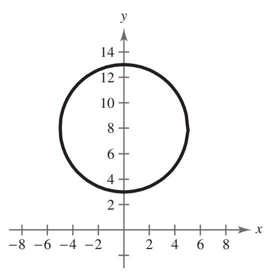


87. $x^2 + (y - 8)^2 = 25$

Circle

Center: $(0, 8)$

Radius: 5



88. $(x - 4)^2 + (y + 7) = 4$

$$(x - 4)^2 = -y - 7 + 4$$

$$(x - 4)^2 = -(y + 3)$$

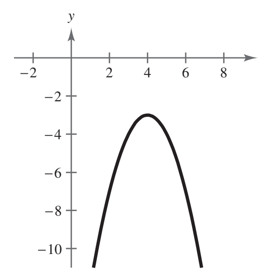
Parabola

Vertex: $(4, -3)$

$P = -\frac{1}{4}$

Focus: $(4, -3.25)$

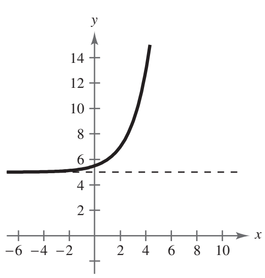
Directrix: $y = -2.75$



89. $f(x) = 2^{x-1} + 5$

Horizontal asymptote: $y = 5$

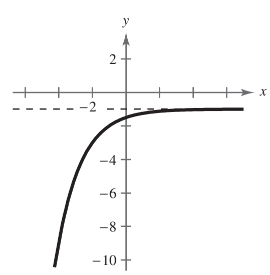
x	-5	-3	-1	0	1	3	5
$f(x)$	5.02	5.06	5.3	5.5	6	9	21



90. $f(x) = -2^{-x-1} - 1$

Horizontal asymptote: $y = -1$

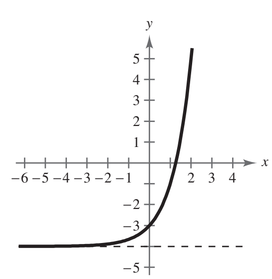
x	-2	-1	0	1	2
$f(x)$	-3	-2	$-\frac{3}{2}$	$-\frac{5}{4}$	$-\frac{9}{8}$



91. $f(x) = 3^x - 4$

Horizontal asymptote: $y = -4$

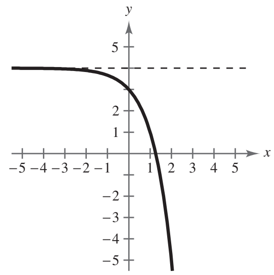
x	-4	-2	-1	0	1	2
$f(x)$	-3.99	-3.89	-3.67	-3	-1	5



92. $f(x) = -3^x + 4$

Horizontal asymptote: $y = 4$

x	-2	-1	0	1	2
$f(x)$	$3\frac{8}{9}$	$3\frac{2}{3}$	3	1	-5



93. Answers will vary.

Review Exercises for Chapter 3

1. $f(x) = 6.1^x$

$f(2.4) = 6.1^{2.4} \approx 76.699$

2. $f(x) = 30^x$

$f(\sqrt{3}) = 30^{\sqrt{3}} = 361.784$

3. $f(x) = 2^{-0.5x}$

$f(\pi) = 2^{-0.5(\pi)} \approx 0.337$

4. $f(x) = 1278^{x/5}$

$f(1) = 1278^{1/5} = 4.181$

5. $f(x) = 7(0.2^x)$

$f(-\sqrt{11}) = 7(0.2^{-\sqrt{11}})$

≈ 1456.529

6. $f(x) = -14(5^x)$

$f(-0.8) = -14(5^{-0.8}) = -3.863$

7. $f(x) = 4^x$

Intercept: $(0, 1)$

Horizontal asymptote: x-axis

Increasing on: $(-\infty, \infty)$

Matches graph (c).

8. $f(x) = 4^{-x}$

Intercept: $(0,1)$

Horizontal asymptote: $y = 0$

Decreasing on: $(-\infty, \infty)$

Matches graph (d).

9. $f(x) = -4^x$

Intercept: $(0, -1)$

Horizontal asymptote: x-axis

Decreasing on: $(-\infty, \infty)$

Matches graph (a).

10. $f(x) = 4^x + 1$

Intercept: $(0, 2)$

Horizontal asymptote: $y = 1$

Increasing on: $(-\infty, \infty)$

Matches graph (b).

11. $f(x) = 5^x$

$g(x) = 5^{x-1}$

Since $g(x) = f(x - 1)$, the graph of g can be obtained by shifting the graph of f one unit to the right.

12. $f(x) = 4^x$, $g(x) = 4^x - 3$

Because $g(x) = f(x) - 3$, the graph of g can be obtained by shifting the graph of f three units downward.

13. $f(x) = \left(\frac{1}{2}\right)^x$

$g(x) = -\left(\frac{1}{2}\right)^{x+2}$

Since $g(x) = -f(x + 2)$, the graph of g can be obtained by reflecting the graph of f about the x-axis and shifting $-f$ two units to the left.

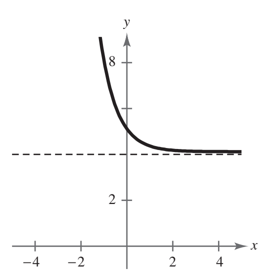
14. $f(x) = \left(\frac{2}{3}\right)^x$, $g(x) = 8 - \left(\frac{2}{3}\right)^x$

Because $g(x) = -f(x) + 8$, the graph of g can be obtained by reflecting the graph of f in the x-axis and shifting the graph of f eight units upward.

15. $f(x) = 4^{-x} + 4$

Horizontal asymptote: $y = 4$

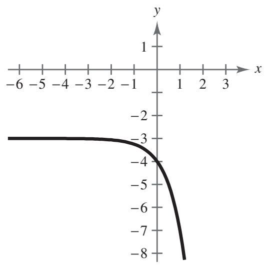
x	-1	0	1	2	3
$f(x)$	8	5	4.25	4.063	4.016



16. $f(x) = -4^x - 3$

Horizontal asymptote: $y = -3$

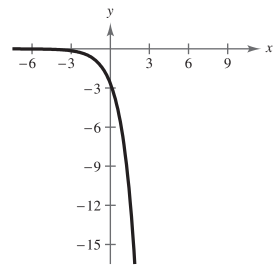
x	-2	-1	0	1	2
$f(x)$	-3.063	-3.25	-4	-7	-19



17. $f(x) = -2.65^{x+1}$

Horizontal asymptote: $y = 0$

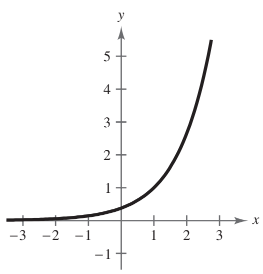
x	-2	-1	0	1	2
$f(x)$	-0.377	-1	-2.65	-7.023	-18.61



18. $f(x) = 2.65^{x-1}$

Horizontal asymptote: $y = 0$

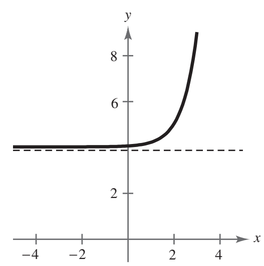
x	-3	-1	0	1	3
$f(x)$	0.020	0.142	0.377	1	7.023



19. $f(x) = 5^{x-2} + 4$

Horizontal asymptote: $y = 4$

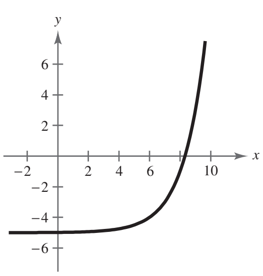
x	-1	0	1	2	3
$f(x)$	4.008	4.04	4.2	5	9



20. $f(x) = 2^{x-6} - 5$

Horizontal asymptote: $y = -5$

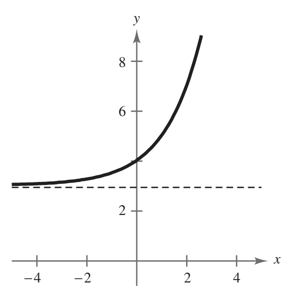
x	0	5	6	7	8	9
$f(x)$	-4.984	-4.5	-4	-3	-1	3



21. $f(x) = \left(\frac{1}{2}\right)^{-x} + 3 = 2^x + 3$

Horizontal asymptote: $y = 3$

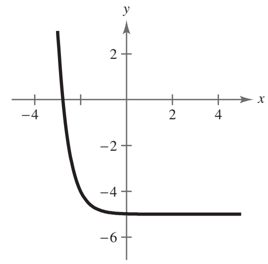
x	-2	-1	0	1	2
$f(x)$	3.25	3.5	4	5	7



22. $f(x) = \left(\frac{1}{8}\right)^{x+2} - 5$

Horizontal asymptote: $y = -5$

x	-3	-2	-1	0	2
$f(x)$	3	-4	-4.875	-4.984	-5



23. $3^{x+2} = \frac{1}{9}$

$3^{x+2} = 3^{-2}$

$x + 2 = -2$

$x = -4$

24. $\left(\frac{1}{3}\right)^{x-2} = 81$

$\left(\frac{1}{3}\right)^{x-2} = 3^4$

$\left(\frac{1}{3}\right)^{x-2} = \left(\frac{1}{3}\right)^{-4}$

$x - 2 = -4$

$x = -2$

25. $e^{5x-7} = e^{15}$

$5x - 7 = 15$

$5x = 22$

$x = \frac{22}{5}$

26. $e^{8-2x} = e^{-3}$

$8 - 2x = -3$

$-2x = -11$

$x = \frac{11}{2}$

27. $e^8 \approx 2980.958$

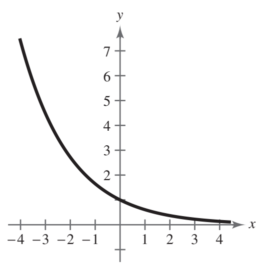
28. $e^{5/8} \approx 1.868$

29. $e^{-1.7} \approx 0.183$

30. $e^{0.278} = 1.320$

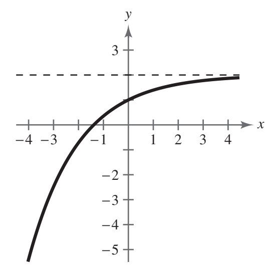
31. $h(x) = e^{-x/2}$

x	-2	-1	0	1	2
$h(x)$	2.72	1.65	1	0.61	0.37



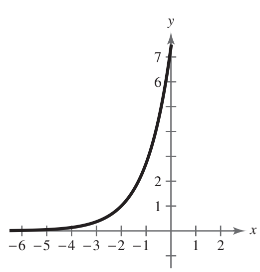
32. $h(x) = 2 - e^{-x/2}$

x	-2	-1	0	1	2
y	-0.72	0.35	1	1.39	1.63



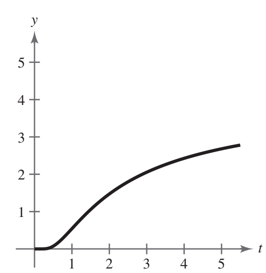
33. $f(x) = e^{x+2}$

x	-3	-2	-1	0	1
$f(x)$	0.37	1	2.72	7.39	20.09



34. $s(t) = 4e^{-2/t}, t > 0$

t	$\frac{1}{2}$	1	2	3	4
y	0.07	0.54	1.47	2.05	2.43



35. $A = 3500\left(1 + \frac{0.065}{n}\right)^{10n}$ or $A = 3500e^{(0.065)(10)}$

n	1	2	4	12	365	Continuous Compounding
A	\$6569.98	\$6635.43	\$6669.46	\$6692.64	\$6704.00	\$6704.39

36. $A = 2000\left(1 + \frac{0.05}{n}\right)^{30n}$ or $A = 2000e^{(0.05)(30)}$

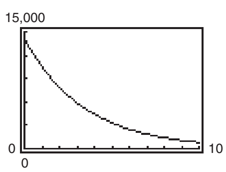
n	1	2	4	12	365	Continuous
A	\$8643.88	\$8799.58	\$8880.43	\$8935.49	\$8962.46	\$8963.38

37. $F(t) = 1 - e^{-t/3}$

(a) $F\left(\frac{1}{2}\right) \approx 0.154$ (b) $F(2) \approx 0.487$ (c) $F(5) \approx 0.811$

38. $V(t) = 14{,}000\left(\frac{3}{4}\right)^t$

(a)



(b) $V(2) = 14{,}000\left(\frac{3}{4}\right)^2 = \$7875$

(c) According to the model, the car depreciates most rapidly at the beginning. Yes, this is realistic.

39. (a) $A = 50{,}000e^{(0.0875)(35)} \approx \$1{,}069{,}047.14$

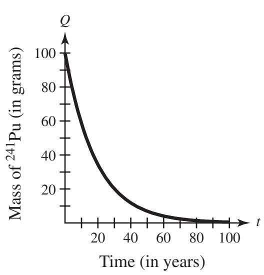
(b) The doubling time is $\frac{\ln 2}{0.0875} \approx 7.9$ years.

40. $Q = 100\left(\frac{1}{2}\right)^{t/14.4}$

(a) For $t = 0$: $Q = 100\left(\frac{1}{2}\right)^{0/14.4} = 100$ grams

(b) For $t = 10$: $Q = 100\left(\frac{1}{2}\right)^{10/14.4} \approx 61.79$ grams

(c)



41. $4^3 = 64$

$\log_4 64 = 3$

42. $25^{3/2} = 125$

$\log_{25} 125 = \frac{3}{2}$

43. $e^{0.8} = 2.2255\ldots$

$\ln 2.2255\ldots = 0.8$

44. $e^0 = 1$

$\ln 1 = 0$

45. $f(x) = \log x$

$f(1000) = \log 1000$

$= \log 10^3 = 3$

46. $\log_9 3 = \log_9 9^{1/2} = \frac{1}{2}$

47. $g(x) = \log_2 x$

$g\left(\frac{1}{8}\right) = \log_2\left(\frac{1}{8}\right) = \log_2 2^{-3} = -3$

48. $f(x) = \log_4 x$

$f\left(\frac{1}{4}\right) = \log_4 \frac{1}{4} = -1$

49. $\log_4(x + 7) = \log_4 14$

$x + 7 = 14$

$x = 7$

50. $\log_8(3x - 10) = \log_8 5$

$3x - 10 = 5$

$3x = 15$

$x = 5$

51. $\ln(x + 9) = \ln 4$

$x + 9 = 4$

$x = -5$

52. $\ln(2x - 1) = \ln(11)$

$2x - 1 = 11$

$2x = 12$

$x = 6$

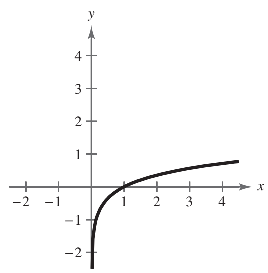
53. $g(x) = \log_7 x \implies x = 7^y$

Domain: $(0, \infty)$

x-intercept: $(1, 0)$

Vertical asymptote: $x = 0$

x	$\frac{1}{7}$	1	7	49
$g(x)$	-1	0	1	2



54. $g(x) = \log_5 x \implies 5^y = x$

Domain: $(0, \infty)$

$\log_5 x = 0$

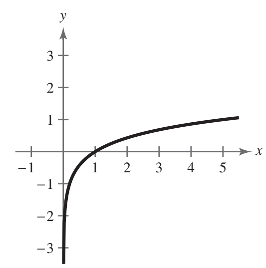
$x = 5^0$

$x = 1$

x-intercept: $(1, 0)$

Vertical asymptote: $x = 0$

x	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25
$g(x)$	-2	-1	0	1	2



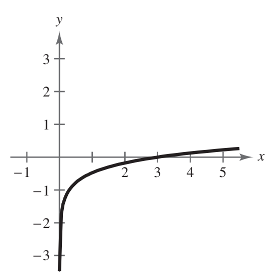
55. $f(x) = \log\left(\dfrac{x}{3}\right) \implies \dfrac{x}{3} = 10^y \implies x = 3(10^y)$

Domain: $(0, \infty)$

x-intercept: $(3, 0)$

Vertical asymptote:
$x = 0$

x	0.03	0.3	3	30
$f(x)$	-2	-1	0	1



56. $f(x) = 6 + \log x$

Domain: $(0, \infty)$

$6 + \log x = 0$

$\log x = -6$

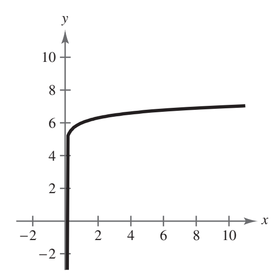
$x = 10^{-6}$

$x = 0.000001$

x-intercept: $(0.000001, 0)$

Vertical asymptote: $x = 0$

x	1	2	4	6	8	10
$f(x)$	6	6.3	6.6	6.8	6.9	7



57. $f(x) = 4 - \log(x + 5)$

Domain: $(-5, \infty)$

x-intercept: $(9995, 0)$

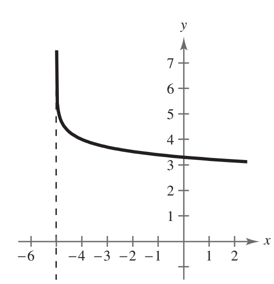
Since $4 - \log(x + 5) = 0 \implies \log(x + 5) = 4$

$x + 5 = 10^4$

$x = 10^4 - 5 = 9995.$

Vertical asymptote: $x = -5$

x	-4	-3	-2	-1	0	1
$f(x)$	4	3.70	3.52	3.40	3.30	3.22



58. $f(x) = \log(x - 3) + 1$

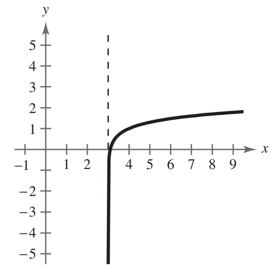
Domain: $(3, \infty)$

$$\begin{aligned} \log(x-3) + 1 &= 0 \\ \log(x-3) &= -1 \\ x - 3 &= 10^{-1} \\ x &= 3.1 \end{aligned}$$

x-intercept: $(3.1, 0)$

Vertical asymptote: $x = 3$

x	4	5	6	7	8
$f(x)$	1	1.3	1.5	1.6	1.7



59. $\ln 22.6 \approx 3.118$

60. $\ln 0.98 \approx -0.020$

61. $\ln e^{-12} = -12$

62. $\ln e^7 = 7$

63. $\ln\left(\sqrt{7} + 5\right) \approx 2.034$

64. $\ln\left(\dfrac{\sqrt{3}}{8}\right) \approx -1.530$

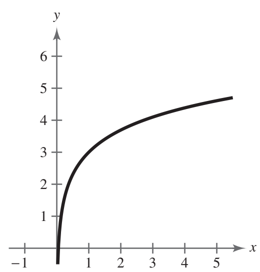
65. $f(x) = \ln x + 3$

Domain: $(0, \infty)$

x-intercept: $\ln x + 3 = 0$

$$\begin{aligned} \ln x &= -3 \\ x &= e^{-3} \\ &(e^{-3}, 0) \end{aligned}$$

Vertical asymptote: $x = 0$



x	1	2	3	$\frac{1}{2}$	$\frac{1}{4}$
$f(x)$	3	3.69	4.10	2.31	1.61

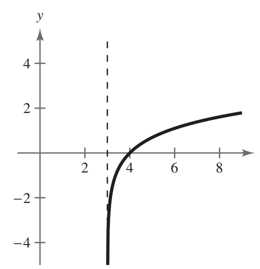
66. $f(x) = \ln(x - 3)$

Domain: $(3, \infty)$

$$\begin{aligned} \ln(x-3) &= 0 \\ x - 3 &= e^0 \\ x &= 4 \end{aligned}$$

x-intercept: $(4, 0)$

Vertical asymptote: $x = 3$



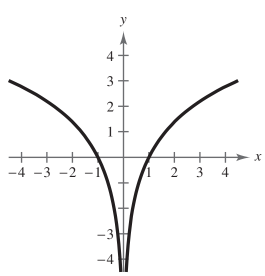
x	3.5	4	4.5	5	5.5
y	-0.69	0	0.41	0.69	0.92

67. $h(x) = \ln(x^2) = 2\ln|x|$

Domain: $(-\infty, 0) \cup (0, \infty)$

x-intercepts: $(\pm 1, 0)$

Vertical asymptote: $x = 0$



x	± 0.5	± 1	± 2	± 3	± 4
y	-1.39	0	1.39	2.20	2.77

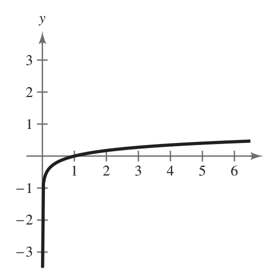
68. $f(x) = \frac{1}{4}\ln x$

Domain: $(0, \infty)$

$$\begin{aligned} \tfrac{1}{4}\ln x &= 0 \\ \ln x &= 0 \\ x &= e^0 \\ x &= 1 \end{aligned}$$

x-intercept: $(1, 0)$

Vertical asymptote: $x = 0$



x	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
y	-0.17	0	0.10	0.17	0.23	0.27

69.

$$\begin{aligned} h &= 116\log(a + 40) - 176 \\ h(55) &= 116\log(55 + 40) - 176 \\ &\approx 53.4 \text{ inches} \end{aligned}$$

70.

$$\begin{aligned} s &= 25 - \frac{13\ln(10/12)}{\ln 3} \\ &\approx 27.16 \text{ miles} \end{aligned}$$

71. $\log_4 9 = \dfrac{\log 9}{\log 4} \approx 1.585$

$\log_4 9 = \dfrac{\ln 9}{\ln 4} \approx 1.585$

72. $\log_{12} 200 = \dfrac{\log 200}{\log 12} \approx 2.132$

$\log_{12} 200 = \dfrac{\ln 200}{\ln 12} \approx 2.132$

73. $\log_{1/2} 5 = \dfrac{\log 5}{\log(1/2)} \approx -2.322$

$\log_{1/2} 5 = \dfrac{\ln 5}{\ln(1/2)} \approx -2.322$

74. $\log_3 0.28 = \dfrac{\log 0.28}{\log 3} \approx -1.159$

$\log_3 0.28 = \dfrac{\ln 0.28}{\ln 3} \approx -1.159$

75. $\log 18 = \log(2 \cdot 3^2)$

$= \log 2 + 2 \log 3$

≈ 1.255

76. $\log_2 \dfrac{1}{12} = \log_2 1 - \log_2 12 = 0 - \log(2^2 \cdot 3)$

$= -2 \log_2 2^2 - \log_2 3 = -2 - \dfrac{\log 3}{\log 2}$

≈ -3.585

77. $\ln 20 = \ln(2^2 \cdot 5)$

$= 2 \ln 2 + \ln 5 \approx 2.996$

78. $\ln 3e^{-4} = \ln 3 + \ln e^{-4}$

$= \ln 3 - 4$

≈ -2.90

79. $\log_5 5x^2 = \log_5 5 + \log_5 x^2$

$= 1 + 2 \log_5 x$

80. $\log_{10} 7x^4 = \log 7 + \log x^4$

$= \log 7 + 4 \log x$

81. $\log_3 \dfrac{6}{\sqrt[3]{x}} = \log_3 6 - \log_3 \sqrt[3]{x}$

$= \log_3(3 \cdot 2) - \log_3 x^{1/3}$

$= \log_3 3 + \log_3 2 - \dfrac{1}{3} \log_3 x$

$= 1 + \log_3 2 - \dfrac{1}{3} \log_3 x$

82. $\log_7 \dfrac{\sqrt{x}}{4} = \log_7 \sqrt{x} - \log_7 4$

$= \log_7 x^{1/2} - \log_7 4$

$= \dfrac{1}{2} \log_7 x - \log_7 4$

83. $\ln x^2y^2z = \ln x^2 + \ln y^2 + \ln z$

$= 2 \ln x + 2 \ln y + \ln z$

84. $\ln 3xy^2 = \ln 3 + \ln x + \ln y^2$

$= \ln 3 + \ln x + 2 \ln y$

85. $\ln\left(\dfrac{x+3}{xy}\right) = \ln(x+3) - \ln xy$

$= \ln(x+3) - [\ln x + \ln y]$

$= \ln(x+3) - \ln x - \ln y$

86. $\ln\left(\dfrac{y-1}{4}\right)^2 = 2 \ln\left(\dfrac{y-1}{4}\right)$

$= 2 \ln(y-1) - 2 \ln 4$

$= 2 \ln(y-1) - \ln 16, \quad y > 1$

87. $\log_2 5 + \log_2 x = \log_2 5x$

88. $\log_6 y - 2 \log_6 z = \log_6 y - \log_6 z^2$

$= \log_6 \dfrac{y}{z^2}$

89. $\ln x - \dfrac{1}{4} \ln y = \ln x - \ln \sqrt[4]{y} = \ln\left(\dfrac{x}{\sqrt[4]{y}}\right)$

90. $3 \ln x + 2 \ln(x+1) = \ln x^3 + \ln(x+1)^2$

$= \ln x^3(x+1)^2$

91. $\dfrac{1}{3} \log_8(x+4) + 7 \log_8 y = \log_8 \sqrt[3]{x+4} + \log_8 y^7$

$= \log_8\left(y^7 \sqrt[3]{x+4}\right)$

92. $-2 \log x - 5 \log(x+6) = \log x^{-2} - \log(x+6)^5$

$= \log \dfrac{x^{-2}}{(x+6)^5}$

$= \log \dfrac{1}{x^2(x+6)^5}$

93. $\frac{1}{2}\ln(2x - 1) - 2\ln(x + 1) = \ln\sqrt{2x - 1} - \ln(x + 1)^2$

$$= \ln\frac{\sqrt{2x - 1}}{(x + 1)^2}$$

94. $5\ln(x - 2) - \ln(x + 2) - 3\ln x = \ln(x - 2)^5 - \ln(x + 2) - \ln x^3$

$$= \ln(x - 2)^5 - [\ln(x + 2) + \ln x^3]$$

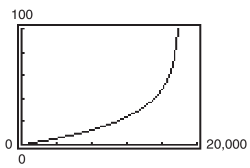
$$= \ln(x - 2)^5 - \ln x^3(x + 2)$$

$$= \ln\frac{(x - 2)^5}{x^3(x + 2)}$$

95. $t = 50\log\frac{18{,}000}{18{,}000 - h}$

(a) Domain: $0 \le h < 18{,}000$

(b)



Vertical asymptote: $h = 18{,}000$

(c) As the plane approaches its absolute ceiling, it climbs at a slower rate, so the time required increases.

(d) $50\log\frac{18{,}000}{18{,}000 - 4000} \approx 5.46$ minutes

96. Using a calculator gives

$s = 84.66 + (-11\ln t)$.

97. $8^x = 512$

$8^x = 8^3$

$x = 3$

98. $6^x = \frac{1}{216}$

$6^x = 6^{-3}$

$x = -3$

99. $e^x = 3$

$x = \ln 3$

100. $e^x = 6$

$\ln e^x = \ln 6$

$x = \ln 6 \approx 1.792$

101. $\log_4 x = 2$

$x = 4^2 = 16$

102. $\log_6 x = -1$

$6^{\log_6 x} = 6^{-1}$

$x = \frac{1}{6}$

103. $\ln x = 4$

$x = e^4$

104. $\ln x = -3$

$x = e^{-3} \approx 0.0498$

105. $e^x = 12$

$\ln e^x = \ln 12$

$x = \ln 12 \approx 2.485$

106. $e^{3x} = 25$

$\ln e^{3x} = \ln 25$

$3x = \ln 25$

$x = \frac{\ln 25}{3} \approx 1.073$

107. $e^{4x} = e^{x^2 + 3}$

$4x = x^2 + 3$

$0 = x^2 - 4x + 3$

$0 = (x - 1)(x - 3)$

$x = 1$ or $x = 3$

108. $14e^{3x+2} = 560$

$e^{3x+2} = 40$

$\ln e^{3x+2} = \ln 40$

$3x + 2 = \ln 40$

$x = \frac{(\ln 40) - 2}{3} \approx 0.563$

109. $2^x + 13 = 35$

$2^x = 22$

$x = \log_2 22$

$= \frac{\log 22}{\log 2}$ or $\frac{\ln 22}{\ln 2}$

$x \approx 4.459$

110. $6^x - 28 = -8$

$6^x = 20$

$\log_6 6^x = \log_6 20$

$x = \log_6 20$

$x = \frac{\ln 20}{\ln 6} \approx 1.672$

111.
$$\begin{aligned} -4(5^x) &= -68 \\ 5^x &= 17 \\ \ln 5^x &= \ln 17 \\ x \ln 5 &= \ln 17 \\ x &= \frac{\ln 17}{\ln 5} \approx 1.760 \end{aligned}$$

112.
$$\begin{aligned} 2(12^x) &= 190 \\ 12^x &= 95 \\ \ln 12^x &= \ln 95 \\ x \ln 12 &= \ln 95 \\ x &= \frac{\ln 95}{\ln 12} \approx 1.833 \end{aligned}$$

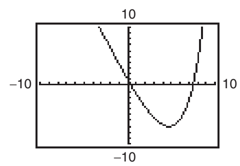
113.
$$e^{2x} - 7e^x + 10 = 0$$
$$(e^x - 2)(e^x - 5) = 0$$
$$e^x = 2 \quad \text{or} \quad e^x = 5$$
$$\ln e^x = \ln 2 \qquad \ln e^x = \ln 5$$
$$x = \ln 2 \approx 0.693 \qquad x = \ln 5 \approx 1.609$$

114.
$$e^{2x} - 6e^x + 8 = 0$$
$$(e^x - 2)(e^x - 4) = 0$$
$$e^x = 2 \quad \text{or} \quad e^x = 4$$
$$x = \ln 2 \qquad x = \ln 4$$
$$x \approx 0.693 \qquad x \approx 1.386$$

115. $2^{0.6x} - 3x = 0$

Graph $y_1 = 2^{0.6x} - 3x$.

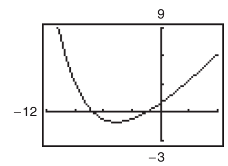
The x-intercepts are at $x \approx 0.392$ and at $x \approx 7.480$.



116. $4^{-0.2x} + x = 0$

Graph $y_1 = 4^{-0.2x} + x$.

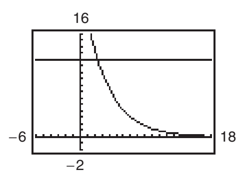
The x-intercepts are at $x \approx -7.038$ and at $x \approx -1.527$.



117. $25e^{-0.3x} = 12$

Graph $y_1 = 25e^{-0.3x}$ and $y_2 = 12$.

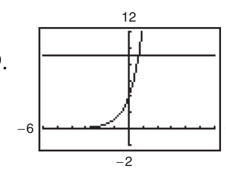
The graphs intersect at $x \approx 2.447$.



118. $4e^{1.2x} = 9$

Graph $y_1 = 4e^{1.2x}$ and $y_2 = 9$.

The graphs intersect at $x \approx 0.676$.



119.
$$\begin{aligned} \ln 3x &= 8.2 \\ e^{\ln 3x} &= e^{8.2} \\ 3x &= e^{8.2} \\ x &= \frac{e^{8.2}}{3} \approx 1213.650 \end{aligned}$$

120.
$$\begin{aligned} \ln 5x &= 7.2 \\ 5x &= e^{7.2} \\ x &= \frac{e^{7.2}}{5} \approx 267.886 \end{aligned}$$

121.
$$\begin{aligned} 2 \ln 4x &= 15 \\ \ln 4x &= \frac{15}{2} \\ e^{\ln 4x} &= e^{7.5} \\ 4x &= e^{7.5} \\ x &= \frac{1}{4}e^{7.5} \approx 452.011 \end{aligned}$$

122.
$$\begin{aligned} 4 \ln 3x &= 15 \\ \ln 3x &= \frac{15}{4} \\ 3x &= e^{15/4} \\ x &= \frac{e^{15/4}}{3} \approx 14.174 \end{aligned}$$

123.
$$\begin{aligned} \ln x - \ln 3 &= 2 \\ \ln \frac{x}{3} &= 2 \\ e^{\ln(x/3)} &= e^2 \\ \frac{x}{3} &= e^2 \\ x &= 3e^2 \approx 22.167 \end{aligned}$$

124.
$$\begin{aligned} \ln\sqrt{x + 8} &= 3 \\ \frac{1}{2}\ln(x + 8) &= 3 \\ \ln(x + 8) &= 6 \\ x + 8 &= e^6 \\ x &= e^6 - 8 \approx 395.429 \end{aligned}$$

125. $\ln\sqrt{x+1} = 2$

$$\frac{1}{2}\ln(x+1) = 2$$
$$\ln(x+1) = 4$$
$$e^{\ln(x+1)} = e^4$$
$$x + 1 = e^4$$
$$x = e^4 - 1 \approx 53.598$$

126. $\ln x - \ln 5 = 4$

$$\ln\frac{x}{5} = 4$$
$$\frac{x}{5} = e^4$$
$$x = 5e^4 \approx 272.991$$

127. $\log_8(x-1) = \log_8(x-2) - \log_8(x+2)$

$$\log_8(x-1) = \log_8\left(\frac{x-2}{x+2}\right)$$
$$x - 1 = \frac{x-2}{x+2}$$
$$(x-1)(x+2) = x - 2$$
$$x^2 + x - 2 = x - 2$$
$$x^2 = 0$$
$$x = 0$$

Since $x = 0$ is not in the domain of $\log_8(x-1)$ or of $\log_8(x-2)$, it is an extraneous solution. The equation has no solution.

128. $\log_6(x+2) - \log_6 x = \log_6(x+5)$

$$\log_6\left(\frac{x+2}{x}\right) = \log_6(x+5)$$
$$\frac{x+2}{x} = x + 5$$
$$x + 2 = x^2 + 5x$$
$$0 = x^2 + 4x - 2$$
$$x = -2 \pm \sqrt{6}, \text{ Quadratic Formula}$$

Only $x = -2 + \sqrt{6} \approx 0.449$ is a valid solution.

129. $\log(1-x) = -1$

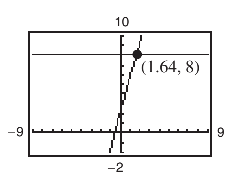
$$1 - x = 10^{-1}$$
$$1 - \tfrac{1}{10} = x$$
$$x = 0.900$$

130. $\log(-x-4) = 2$

$$-x - 4 = 10^2$$
$$-x = 100 + 4$$
$$x = -104$$

131. $2\ln(x+3) + 3x = 8$

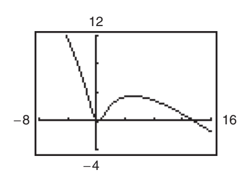
Graph $y_1 = 2\ln(x+3) + 3x$ and $y_2 = 8$.



The graphs intersect at approximately $(1.643, 8)$. The solution of the equation is $x \approx 1.643$.

132. $6\log(x^2+1) - x = 0$

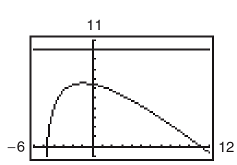
Graph $y_1 = 6\log(x^2+1) - x$.



The x-intercepts are at $x = 0$, $x \approx 0.416$, and $x \approx 13.627$.

133. $4\ln(x+5) - x = 10$

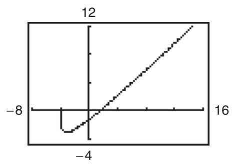
Graph $y_1 = 4\ln(x+5) - x$ and $y_2 = 10$.



The graphs do not intersect. The equation has no solution.

134. $x - 2\log(x + 4) = 0$

Let $y_1 = x - 2\log(x + 4)$.



The x-intercepts are at $x \approx -3.990$ and $x \approx 1.477$.

135. $3(7550) = 7550e^{0.0725t}$

$$3 = e^{0.0725t}$$

$$\ln 3 = \ln e^{0.0725t}$$

$$\ln 3 = 0.0725t$$

$$t = \frac{\ln 3}{0.0725} \approx 15.2 \text{ years}$$

136. $S = 93\log(d) + 65$

$$283 = 93\log(d) + 65$$

$$218 = 93\log(d)$$

$$\log(d) = \frac{218}{93}$$

$$d = 10^{(218/93)} \approx 220.8 \text{ miles}$$

137. $y = 3e^{-2x/3}$

Exponential decay model

Matches graph (e).

138. $y = 4e^{2x/3}$

Exponential growth model

Matches graph (b).

139. $y = \ln(x + 3)$

Logarithmic model

Vertical asymptote: $x = -3$

Graph includes $(-2, 0)$

Matches graph (f).

140. $y = 7 - \log(x + 3)$

Logarithmic model

Vertical asymptote: $x = -3$

Matches graph (d).

141. $y = 2e^{-(x+4)^2/3}$

Gaussian model

Matches graph (a).

142. $y = \dfrac{6}{1 + 2e^{-2x}}$

Logistics growth model

Matches graph (c).

143. $y = ae^{bx}$

$$2 = ae^{b(0)} \implies a = 2$$

$$3 = 2e^{b(4)}$$

$$1.5 = e^{4b}$$

$$\ln 1.5 = 4b \implies b \approx 0.1014$$

Thus, $y \approx 2e^{0.1014x}$.

144. $y = ae^{bx}$

$$\frac{1}{2} = ae^{b(0)} \implies a = \frac{1}{2}$$

$$5 = \frac{1}{2}e^{b(5)}$$

$$10 = e^{5b}$$

$$\ln 10 = 5b$$

$$\frac{\ln 10}{5} = b$$

$$b \approx 0.4605$$

$$y = \frac{1}{2}e^{0.4605x}$$

145. $P = 3499e^{0.0135t}$

$$4.5 \text{ million} = 4500 \text{ thousand}$$
$$4500 = 3499e^{0.0135t}$$
$$\frac{4500}{3499} = e^{0.0135t}$$
$$\ln\left(\frac{4500}{3499}\right) = 0.0135t$$
$$t = \frac{\ln(4500/3499)}{0.0135} \approx 18.6 \text{ years}$$

According to this model, the population of South Carolina will reach 4.5 million during the year 2008.

146. $y = Ce^{kt}$

$$\frac{1}{2}C = Ce^{(250{,}000)k}$$
$$\ln\frac{1}{2} = \ln e^{(250{,}000)k}$$
$$\ln\frac{1}{2} = 250{,}000k$$
$$k = \frac{\ln(1/2)}{250{,}000}$$

When $t = 5000$, we have

$$y = Ce^{[\ln(1/2)/250{,}000](5000)} \approx 0.986C = 98.6\%C.$$

After 5000 years, approximately 98.6% of the radioactive uranium II will remain.

147. (a) $20{,}000 = 10{,}000e^{r(5)}$

$$2 = e^{5r}$$
$$\ln 2 = 5r$$
$$\frac{\ln 2}{5} = r$$
$$r \approx 0.138629$$
$$= 13.8629\%$$

(b) $A = 10{,}000e^{0.138629}$

$$\approx \$11{,}486.98$$

148. $N_0 = 2000$ and $N_3 = 1400$ so $N = 2000e^{kt}$ and:

$$1400 = 2000e^{3k}$$
$$\frac{7}{10} = e^{3k}$$
$$3k = \ln\left(\frac{7}{10}\right)$$
$$k = \frac{\ln(7/10)}{3} = -0.11889$$

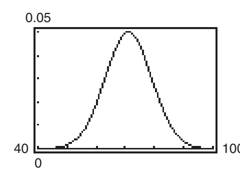
The population one year ago:

$$N(4) = 2000e^{-0.11889(4)}$$
$$= 1243 \text{ bats}$$

149. $y = 0.0499e^{-(x-71)^2/128}$,

$40 \le x \le 100$

(a) Graph $y_1 = 0.0499e^{-(x-71)^2/128}$.



(b) The average test score is 71.

150. $N = \dfrac{157}{1 + 5.4e^{-0.12t}}$

(a) When $N = 50$:

$$50 = \frac{157}{1 + 5.4e^{-0.12t}}$$
$$1 + 5.4e^{-0.12t} = \frac{157}{50}$$
$$5.4e^{-0.12t} = \frac{107}{50}$$
$$e^{-0.12t} = \frac{107}{270}$$
$$-0.12t = \ln\frac{107}{270}$$
$$t = \frac{\ln(107/270)}{-0.12} \approx 7.7 \text{ weeks}$$

(b) When $N = 75$:

$$75 = \frac{157}{1 + 5.4e^{-0.12t}}$$
$$1 + 5.4e^{-0.12t} = \frac{157}{75}$$
$$5.4e^{-0.12t} = \frac{82}{75}$$
$$e^{-0.12t} = \frac{82}{405}$$
$$-0.12t = \ln\frac{82}{405}$$
$$t = \frac{\ln(82/405)}{-0.12} \approx 13.3 \text{ weeks}$$

151. $\beta = 10 \log\left(\frac{I}{10^{-16}}\right)$

$125 = 10 \log\left(\frac{I}{10^{-16}}\right)$

$12.5 = \log\left(\frac{I}{10^{-16}}\right)$

$10^{12.5} = \frac{I}{10^{-16}}$

$I = 10^{-3.5}$ watt/cm^2

152. $R = \log I$ since $I_0 = 1$.

(a) $\log I = 8.4$

$I = 10^{8.4} \approx 251{,}188{,}643$

(b) $\log I = 6.85$

$I = 10^{6.85} \approx 7{,}079{,}458$

(c) $\log I = 9.1$

$I = 10^{9.1} \approx 1{,}258{,}925{,}412$

153. True. By the inverse properties, $\log_b b^{2x} = 2x$.

154. False. $\ln x + \ln y = \ln(xy) \neq \ln(x + y)$

155. Since graphs (b) and (d) represent exponential decay, b and d are negative.

Since graph (a) and (c) represent exponential growth, a and c are positive.

Problem Solving for Chapter 3

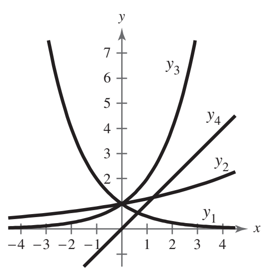
1. $y = a^x$

$y_1 = 0.5^x$

$y_2 = 1.2^x$

$y_3 = 2.0^x$

$y_4 = x$



The curves $y = 0.5^x$ and $y = 1.2^x$ cross the line $y = x$. From checking the graphs it appears that $y = x$ will cross $y = a^x$ for $0 \le a \le 1.44$.

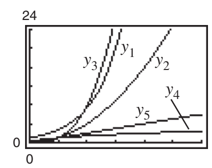
2. $y_1 = e^x$

$y_2 = x^2$

$y_3 = x^3$

$y_4 = \sqrt{x}$

$y_5 = |x|$



The function that increases at the fastest rate for "large" values of x is $y_1 = e^x$. (*Note:* One of the intersection points of $y = e^x$ and $y = x^3$ is approximately (4.536, 93) and past this point $e^x > x^3$. This is not shown on the graph above.)

3. The exponential function, $y = e^x$, increases at a faster rate than the polynomial function $y = x^n$.

4. It usually implies rapid growth.

5. (a) $f(u + v) = a^{u+v}$

$= a^u \cdot a^v$

$= f(u) \cdot f(v)$

(b) $f(2x) = a^{2x}$

$= (a^x)^2$

$= [f(x)]^2$

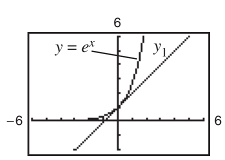
6. $[f(x)]^2 - [g(x)]^2 = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$

$= \left(\frac{e^{2x} + 2 + e^{-2x}}{4}\right) - \left(\frac{e^{2x} - 2 + e^{-2x}}{4}\right)$

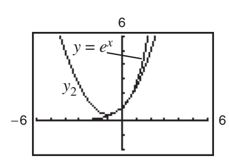
$= \frac{4}{4}$

$= 1$

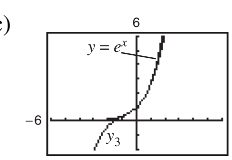
7. (a)



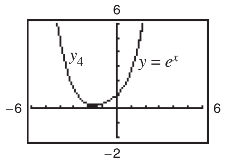
(b)



(c)



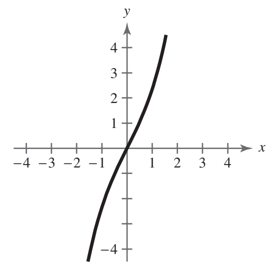
8. $y_4 = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!}$



As more terms are added, the polynomial approaches e^x.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

9.
$$\begin{aligned} f(x) &= e^x - e^{-x} \\ y &= e^x - e^{-x} \\ x &= e^y - e^{-y} \\ x &= \frac{e^{2y} - 1}{e^y} \\ xe^y &= e^{2y} - 1 \end{aligned}$$



$$e^{2y} - xe^y - 1 = 0$$

$$e^y = \frac{x \pm \sqrt{x^2 + 4}}{2} \qquad \text{Quadratic Formula}$$

Choosing the positive quantity for e^y we have

$y = \ln\left(\dfrac{x + \sqrt{x^2 + 4}}{2}\right)$. Thus, $f^{-1}(x) = \ln\left(\dfrac{x + \sqrt{x^2 + 4}}{2}\right)$.

10. $f(x) = \dfrac{a^x + 1}{a^x - 1}$, $a > 0$, $a \neq 1$

$$\begin{aligned} x &= \frac{a^y + 1}{a^y - 1} \\ x(a^y - 1) &= a^y + 1 \\ xa^y - a^y &= x + 1 \\ a^y(x - 1) &= x + 1 \\ a^y &= \frac{x + 1}{x - 1} \end{aligned}$$

$$y = \log_a\left(\frac{x + 1}{x - 1}\right) = \frac{\ln\left(\dfrac{x + 1}{x - 1}\right)}{\ln a} = f^{-1}(x)$$

11. Answer (c). $y = 6(1 - e^{-x^2/2})$

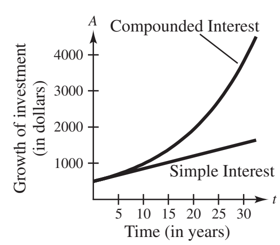
The graph passes through $(0, 0)$ and neither (a) nor (b) pass through the origin. Also, the graph has y-axis symmetry and a horizontal asymptote at $y = 6$.

12. (a) The steeper curve represents the investment earning compound interest, because compound interest earns more than simple interest. With simple interest there is no compounding so the growth is linear.

(b) Compound interest formula: $A = 500\left(1 + \frac{0.07}{1}\right)^{(1)t} = 500(1.07)^t$

Simple interest formula: $A = Prt + P = 500(0.07)^t + 500$

(c) One should choose compound interest since the earnings would be higher.



13. $y_1 = c_1\left(\dfrac{1}{2}\right)^{t/k_1}$ and $y_2 = c_2\left(\dfrac{1}{2}\right)^{t/k_2}$

$$\begin{aligned} c_1\left(\frac{1}{2}\right)^{t/k_1} &= c_2\left(\frac{1}{2}\right)^{t/k_2} \\ \frac{c_1}{c_2} &= \left(\frac{1}{2}\right)^{(t/k_2 - t/k_1)} \\ \ln\left(\frac{c_1}{c_2}\right) &= \left(\frac{t}{k_2} - \frac{t}{k_1}\right)\ln\left(\frac{1}{2}\right) \\ \ln c_1 - \ln c_2 &= t\left(\frac{1}{k_2} - \frac{1}{k_1}\right)\ln\left(\frac{1}{2}\right) \\ t &= \frac{\ln c_1 - \ln c_2}{[(1/k_2) - (1/k_1)]\ln(1/2)} \end{aligned}$$

14. $B = B_0a^{kt}$ through $(0, 500)$ and $(2, 200)$

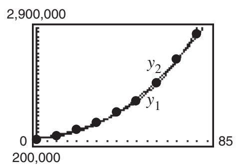
$$\begin{aligned} B_0 &= 500 \\ 200 &= 500a^{k(2)} \\ \frac{2}{5} &= a^{2k} \\ \log_a\left(\frac{2}{5}\right) &= 2k \\ \frac{1}{2}\log_a\left(\frac{2}{5}\right) &= k \end{aligned}$$

$$B = 500a^{[(1/2)\log_a(2/5)]t} = 500[a^{\log_a(2/5)}]^{t/2} = 500\left(\frac{2}{5}\right)^{t/2}$$

15. (a) $y \approx 252.606(1.0310)^t$

(b) $y \approx 400.88t^2 - 1464.6t + 291{,}782$

(c)



(d) Both models appear to be "good fits" for the data, but neither would be reliable to predict the population of the United States in 2010. The exponential model approaches infinity rapidly.

16. Let $\log_a x = m$ and $\log_{a/b} x = n$. Then $x = a^m$ and $x = (a/b)^n$.

$$a^m = \left(\frac{a}{b}\right)^n$$

$$a^{m/n} = \frac{a}{b}$$

$$a^{m/n-1} = \frac{1}{b}$$

$$\log_a \frac{1}{b} = \frac{m}{n} - 1$$

$$1 + \log_a \frac{1}{b} = \frac{m}{n}$$

$$1 + \log_a \frac{1}{b} = \frac{\log_a x}{\log_{a/b} x}$$

17.
$$(\ln x)^2 = \ln x^2$$
$$(\ln x)^2 - 2\ln x = 0$$
$$\ln x(\ln x - 2) = 0$$
$$\ln x = 0 \quad \text{or} \quad \ln x = 2$$
$$x = 1 \quad \text{or} \quad x = e^2$$

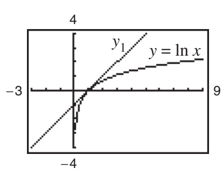
18. $y = \ln x$

$y_1 = x - 1$

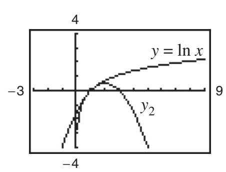
$y_2 = (x-1) - \frac{1}{2}(x-1)^2$

$y_3 = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

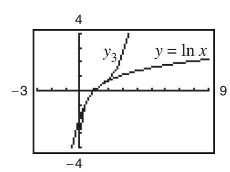
(a)



(b)



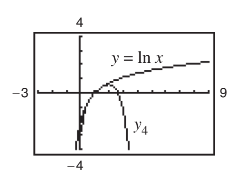
(c)



19. $y^4 = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$

The pattern implies that

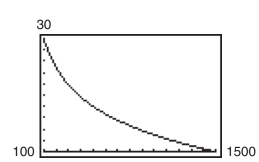
$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \ldots.$



20.

$y = ab^x$	$y = ax^b$
$\ln y = \ln(ab^x)$	$\ln y = \ln(ax^b)$
$\ln y = \ln a + \ln b^x$	$\ln y = \ln a + \ln x^b$
$\ln y = \ln a + x\ln b$	$\ln y = \ln a + b\ln x$
$\ln y = (\ln b)x + \ln a$	$\ln y = b\ln x + \ln a$
Slope: $m = \ln b$	Slope: $m = b$
y-intercept: $(0, \ln a)$	y-intercept: $(0, \ln a)$

21. $y = 80.4 - 11\ln x$



$y(300) = 80.4 - 11\ln 300 \approx 17.7\ \text{ft}^3/\text{min}$

22. (a) $\dfrac{450}{30} = 15$ cubic feet per minute

(b) $$\begin{aligned} 15 &= 80.4 - 11 \ln x \\ 11 \ln x &= 65.4 \\ \ln x &= \frac{65.4}{11} \\ x &= e^{65.4/11} \\ x &\approx 382 \text{ cubic feet of air space per child.} \end{aligned}$$

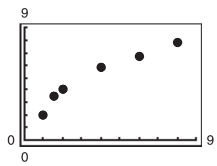
(c) Total air space required: $382(30) = 11{,}460$ cubic feet

Let $x =$ floor space in square feet and $h = 30$ feet.

$$\begin{aligned} V &= xh \\ 11{,}460 &= x(30) \\ x &= 382 \end{aligned}$$

If the ceiling height is 30 feet, the minimum number of square feet of floor space required is 382 square feet.

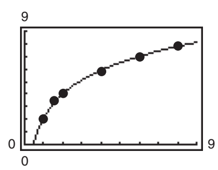
23. (a)



(b) The data could best be modeled by a logarithmic model.

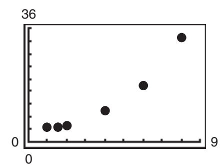
(c) The shape of the curve looks much more logarithmic than linear or exponential.

(d) $y \approx 2.1518 + 2.7044 \ln x$



(e) The model is a good fit to the actual data.

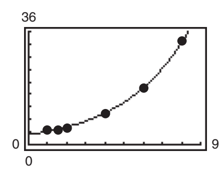
24. (a)



(b) The data could best be modeled by an exponential model.

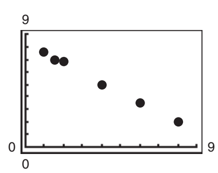
(c) The data scatter plot looks exponential.

(d) $y \approx 3.114(1.341)^x$



(e) The model graph hits every point of the scatter plot.

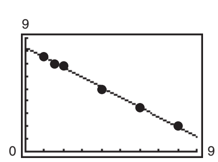
25. (a)



(b) The data could best be modeled by a linear model.

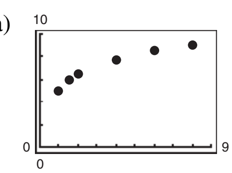
(c) The shape of the curve looks much more linear than exponential or logarithmic.

(d) $y \approx -0.7884x + 8.2566$



(e) The model is a good fit to the actual data.

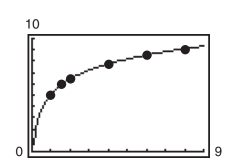
26. (a)



(b) The data could best be modeled by a logarithmic model.

(c) The data scatter plot looks logarithmic.

(d) $y \approx 5.099 + 1.92 \ln(x)$



(e) The model graph hits every point of the scatter plot.

Chapter 3 Practice Test

1. Solve for x: $x^{3/5} = 8$.

2. Solve for x: $3^{x-1} = \frac{1}{81}$.

3. Graph $f(x) = 2^{-x}$.

4. Graph $g(x) = e^x + 1$.

5. If \$5000 is invested at 9% interest, find the amount after three years if the interest is compounded

(a) monthly. (b) quarterly. (c) continuously.

6. Write the equation in logarithmic form: $7^{-2} = \frac{1}{49}$.

7. Solve for x: $x - 4 = \log_2 \frac{1}{64}$.

8. Given $\log_b 2 = 0.3562$ and $\log_b 5 = 0.8271$, evaluate $\log_b \sqrt[4]{8/25}$.

9. Write $5 \ln x - \frac{1}{2} \ln y + 6 \ln z$ as a single logarithm.

10. Using your calculator and the change of base formula, evaluate $\log_9 28$.

11. Use your calculator to solve for N: $\log_{10} N = 0.6646$

12. Graph $y = \log_4 x$.

13. Determine the domain of $f(x) = \log_3(x^2 - 9)$.

14. Graph $y = \ln(x - 2)$.

15. True or false: $\dfrac{\ln x}{\ln y} = \ln(x - y)$

16. Solve for x: $5^x = 41$

17. Solve for x: $x - x^2 = \log_5 \frac{1}{25}$

18. Solve for x: $\log_2 x + \log_2(x - 3) = 2$

19. Solve for x: $\dfrac{e^x + e^{-x}}{3} = 4$

20. Six thousand dollars is deposited into a fund at an annual interest rate of 13%. Find the time required for the investment to double if the interest is compounded continuously.